Journal of Engineering Sciences
Vol 10, Issue 10, Oct /2019
ISSN NO:0377-9254

A COMBINATION OF CONVERSION FOR NUMERICAL COMBINATION OF SINGULAR INTEGRALS IN THE BEM

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ABSTRACT:

We present a nonlinear composite transformation for polynomial transformation (PT) and sigmoid transformation (ST) as CT PS (composite transformation with polynomial and sigmoidal transitions). This new transformation can be used both for Cauchy central value integrations (CPV) and for individual singular integrals with individual internal characteristics. In addition, numerical evaluation using CT PS should be more efficient than using PT or ST alone. With numerical examples, we show that the CT PS method presented, with the Gauss-Legendre quadrature rule, effectively improves the rounding errors of the current methods. In particular, the results of the current method have proven to be excellent for CPV integrations with proximal endpoint differences.

Keywords: weakly singular integral; CPV integral; Gauss–Legendre quadrature rule; polynomial/sigmoidal transformation

1. INTRODUCTION:

In the boundary element method (BEM), the dominant coefficients resulting from the discretization of the singular boundary integral representation of the field variable inevitably consist of singular integrals [1–3]. Thus, the accurate numerical evaluation of singular integrals is very important to reliable implementation of the BEM. In this paper we consider CPV integrals and weakly singular integrals appearing in the usual BEM in two-dimensional problems of the form Among the numerical methods for evaluation of CPV or weakly singular integrals, polynomial transformation (PT) methods [4–8] and sigmoidal transformation (ST) methods [9] appear to be the most attractive due to their simplicity and generality. All of these non-linear transformations have a property that the Jacobian of the transformation is zero at the singular point, which renders the weakly singular kernel into a regular one. Another merit of these transformations is that the standard quadrature rule with them uses the same initial integration points and weights as those used for the regular integrals. Telles [4] rstly proposed a non-linear transformation technique for the evaluation of weakly and quasi-singular integrals. This includes a quadratic PT for the integrals with end point singularity and a cubic PT for the integrals with interior point singularity. Cerrolaza and Alarcon [6] presented a bi-cubic PT technique which is applicable to weakly singular integrals as well as CPV integrals. Mathematical foundations of application of general PT to CPV and weakly singular integrals were detailed by Doblar e and Gracia [8]. Very recently, for evaluating weakly singular integrals, the ST, which is a special kind of the non-linear transformation, has been used actively [9]. With respect to weakly singular integrals, it has been reported that the ST is prominent among the traditional non-linear transformation techniques. The accuracy of the ST technique is due to the primary property that it clusters the integration points on the interval towards the end points rapidly. Furthermore, Johnston and Elliott have obtained asymptotic estimates of the truncation errors for the Gauss–Legendre quadrature rule with the ST, which implies the superiority in both theoretical analysis and numerical results. In addition, it was shown that a semi-ST technique can improve a standard ST technique for weakly singular integrals possessing only one end point singularity by using exponential functions, have proposed new kinds of ST containing an additional parameter. It has been shown that, by controlling the parameter, one can obtain su ciently accurate numerical evaluations of weakly singular integrals with end point singularities. Using a non-linear transformation like the ST for the weakly singular integral with an interior singularity, the integral should be split into two with end point singularities. In the result, the number of function evaluations is twice the number of integration points as is Sato's PT [5] or the bi-cubic PT [6]. On the other hand, Singh and Tanaka presented a comparison of the smoothing property, numerical convergence and accuracy of the available non-linear transformation methods including the transformations of Telles [4], Sato et al. [5], Doblar e and Gracia [8], and Johnston [12]. They investigated eectiveness of generalized non-linear transformations valid for any type and location of singularity. Even if the PT and ST techniques mentioned above have respective advantages, they also have some aws or limits in general as follows: (i) E ciency of the PT techniques suitable for CPV integrals is not equally sustained for weakly singular integrals. (ii) A single non-linear transformation (either PT or ST), without partitioning the integration interval, is not e cient for weakly singular integrals involving interior singularities. (iii) The ST technique is not applicable to the case of CPV integrals directly. (iv) The approximation errors of both PT and ST techniques are not satisfactory when the singular point is located near an end of the integration interval. In this work, based on the observations above, we propose a composite transformation with a Doblar e's polynomial transformation [8] and a sigmoidal transformation (CT PS). The objective of the presented CT PS is to be available for both CPV and weakly singular integrals involving interior singularity without any split of the integration interval. This saves the number of computations in evaluating singular integrals with interior singularities. In addition, a new ST proposed by is introduced in this work to improve the capacity of CT PS when the interior singularity is located in the vicinity of the end point. In the result, as one can identify from results of the numerical examples in the last section, the present technique with CT PS may considerably overcome the problems (i)–(iv) given above. In practice, for some numerical examples in this paper, it is shown that Gauss–Legendre quadrature rule after CT PS is available for both CPV and weakly singular integrals. Especially, e ciency of the present method is remarkable when the interior singular point of the CPV integral is close to an end of the integration interval. A brief overview of the general ST is given and then examples of the traditional ST and a parametric ST are introduced in Section 2. In Section 3, the focus of this work, CT PS is developed and some functional properties are summarized. Section 4 treats numerical examples to investigate the availability of the present method.

2. SIGMOIDAL TRANSFORMATIONS

A real valued function m(x) is said to be a sigmoidal transformation of order m ∈ N if the following conditions are satised:

m(x)∈ C1[0; 1]∩ C∞(0; 1).

(ii) m(x) + m(1 − x) = 1, 06x61.

(iii) m(x) is strictly increasing on [0; 1] with m(0) = 0.

(iv) m(x) is strictly increasing on [0; 1 2] with m(0) = 0.

(v) (j) m (x) = O(xm−j) near x =0 (j = 0; 1; 2;:::;m).

The ST m(x) has some general properties as given in the following theorem.

A sigmoidal transformation m(x), m¿2, satises the properties:

m(0) = 0, m(1=2) = 1=2, m(1) = 1

(II) Near x = 1, (j) m (x) = 0; j + O((1 − x)m−j); j = 0; 1; 2;:::;m

where 0; j is Kronecher's delta

m(x) = m(1 − x), 06x61, so that m(0) = m(1) = 0 In general, for 0¡x¡1, (j) m (x)+(−1)j (j) m (1 − x)=0; j = 1; 2; 3;:::

If m(x)∈ C∞[0; 1], then (j) m (0) = (j) m (1) = 0; for any integer 16j¡m (j) m (1=2) = 0; for any even integer 26j¡m

3. COMPOSITE TRANSFORMATIONS WITH POLYNOMIAL AND SIGMOIDAL TRANSFORMATIONS

If we simply modify a fourth degree PT of Doblar e and Gracia [8] as (t) = xs(1 − t 4) + t 3 ; −1¡t¡1 then it has important properties as (P1) (0) = xs, (1) = 1, (−1) = −1 (P2) (0) = 0, (0) = 0 with (t) = −4xst 3 + 3t 2. If one takes a non-linear transformation satisfying (P1) and (P2), like (8), then CPV integrals in (1) become in which g(t) = f((t)). If g(t) is H older continuous on the interval containing the singular point t = 0 then I1 is only weakly singular and, if g(t) is continuous, regular. In any case, I1 can be computed accurately because the dierence between the two innities is multiplied by (0) = 0. Since the local behaviour of (t) and (t) − (0) near the singular point t = 0 is

(t) = (0) + (0)t + 1 2!(0)t 2 + ··· (t) − (0) = (0)t + 1 2!(0)t 2 + 1 3!(0)t 3 + ···. In which l is the rst non-vanishing derivative of (t) at t = 0 and A(t) is a bounded function. Then the CPV integral in (11) vanishes analytically and, in this case, also vanishes if a standard Gauss quadrature rule with an even number of integration points is used [4]. On the other hand, the logarithmic singular integral J in (2) becomes. The singularity of the integrand O(log |x − xs|) has been weakened as O(t 2 log |t|) by the non-linear transformation (t) in (8). The facts mentioned above make it possible to expect good numerical evaluation of the singular integrals in the form of (1) and (2). However, one can observe in Doblare and Gracia [8] that the improvement of the numerical evaluation of CPV integrals, by the nonlinear transformation like (8), is not eective when the singular point xs is near the end of the boundary element. Moreover, as mentioned previously, the use of a PT suitable for CPV integrals is not equally eective for weakly singular integrals. These are the motives of the present work. Using an ST, m(x) of order m¿2, we dene a non-linear transformation, m; k (t) from the interval [−1; 1] onto itself, as

4. CONCLUSION:

We have presented a composite transformation (CT PS) technique, using a fourth degree PT [8] and an

ST, for the numerical evaluation of CPV and weakly singular integrals. This new composite transformation contains primary properties of both PT and ST, which implies that it is protable for both CPV and weakly singular integrals. Furthermore, the CT PS technique does not require any split in the integration interval. Comparison of the CT PS technique with the existing PT or ST technique for some numerical examples has exhibited that the former can overcome most weaknesses of the PT or ST method described in Section 1. Observing the numerical results of the examples, we summarize this paper as follows:

(i) The CT PS technique is available for both CPV and weakly singular integrals involving interior singularities.

(ii) For CPV integrals with near end singular points, the improvement of the errors by the CT PS technique is prominent. For weakly singular integrals, the existing techniques such as Sato's PT and semi-ST ones are superior with respect to the accuracy. Nevertheless, the CT PS technique without the splitting of the integration interval is advantageous so long as the tolerance error is not exceedingly small.

(iii) In choosing an ST to compose CT PS, the ST of order 2 (the lowest order) is most adequate. In particular, when the singular point of the CPV integral is near to an end of the integration interval, the ST Y 2 (b; t) in (6) is recommended as it can better the error su ciently by selecting a proper value of the parameter, b.

REFERENCES:

1. Brebbia CA, Telles JCF, Wrobel LC. Boundary Element Techniques. Springer: Berlin, 1984.

2. Brebbia CA, Dominguez J. Boundary Elements An Introductory Course. Computational Mechanics Publications: Southampton, 1992.

3. Banerjee PK. The Boundary Element Methods In Engineering. McGraw-Hill: London, 1994.

4. Telles JCF. A self-adaptive co-ordinate transformation for e cient numerical evaluation of general boundary element integrals. International Journal for Numerical Methods in Engineering 1987; 24:959–973.

5. Sato M, Yoshiyoka S, Tsukui K, Yuuki R. Accurate numerical integration of singular kernels in the twodimensional boundary element method. In Boundary Elements X , vol. 1, Brebbia CA (ed). Springer: Berlin, 1988; 279–296.

6. Cerrolaza M, Alarcon E. A bi-cubic transformation for the numerical evaluation of the Cauchy principal value integrals in boundary elements. International Journal for Numerical Methods in Engineering 1989; 28: 987–999.

7. Sanz Serna JM, Doblar e M, Alarcon E. Remarks on methods for the computation of boundary-element integrals by co-ordinate transformation. Communications in Applied Numerical Methods 1990; 6:121–123.

8. Doblar e M, Gracia L. On non-linear transformations for the integration of weekly-singular and Cauchy principal value integrals. International Journal for Numerical Methods in Engineering 1997; 40:3325–3358.

9. Sidi A. A new variable transformation for numerical integration. In Numerical Integration, vol. IV, Brass H, Hammerlin G (eds). Birkhauser: Basel, 1993; 359–373.

10. Elliott D. The cruciform crack problem and sigmoidal transformations. Mathematical Methods in the Applied Sciences 1997; 20:121–132.

11. Elliott D. Sigmoidal transformations and the trapezoidal rule. Journal of the Australian Mathematical Society Series B 1998; 40(E):E77–E137.

12. Johnston PR. Application of sigmoidal transformations to weakly singular and neer singular boundary element integrals. International Journal for Numerical Methods in Engineering 1999; 45:1333–1348.