Journal of Engineering Sciences
Vol 10, Issue 11, NOV/2019
ISSN NO:0377-9254

Nano Minimal and Maximal $\mathcal{M}$-open Sets

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Abstract

In this paper, we introduce the concepts of new class of sets namely nano minimal $\mathcal{M}$-open and nano maximal $\mathcal{M}$-open sets. Several properties of these new notions are investigated and the connections between them are studied.

Keywords and phrases: $\mathfrak{N}$Mi$\mathcal{M}$o, $\mathfrak{N}$Mi$\mathcal{M}$c, $\mathfrak{N}$Ma$\mathcal{M}$o and $\mathfrak{N}$Ma$\mathcal{M}$c sets.
AMS (2000) subject classification: 54B05.

1 Introduction and Preliminaries

Lellis Thivagar [8] introduced the notion of Nano topology (briefly, $\mathfrak{N}\mathfrak{T}$) by using theory approximations and boundary region of a subset of an universe in terms of an equivalence relation on it and also defined Nano closed (briefly, $\mathfrak{N}$c) sets , Nano-interior (briefly, $\mathfrak{N}$int) and Nano-closure (briefly, $\mathfrak{N}$cl) in a nano topological spaces (briefly, $\mathfrak{N}$ts). Carmel [13] discussed some weak forms of $\mathfrak{N}$o sets and $\mathfrak{N}\theta$ open (briefly, $\mathfrak{N}\theta$o) sets. The notion of M-open sets in topological spaces were introduced by El-Maghrabi and Al-Juhani [3] in 2011 and studied some of their properties. The class of sets namely, M-open sets are playing more important role in topological spaces, because of their applications in various fields of Mathematics and other real fields. Now a days topological approaches are being investigated in a big way in various diverse field such as computer graphics, evolutionary theory, robotics etc.[4, 6, 14] to name a few. One such approach to computer graphics utilizes finite, connected order topological space [5]. In a finite topological space, the intersection of all open neighbourhoods of a point p is again an open neighbourhood of p, which is the smallest one. It is called the minimal neighbourhood of p. The topology of a finite space is completely determined by its minimal neighbourhoods. However, in a general framework of all topological spaces this is not true. Nevertheless, the sets which are realized as arbitrary intersection of open sets in topology are quite interesting. In this paper, we have made an investigation of all these type of sets. The minimal open sets, as we call them, being a weaker form of open sets, are studied here in the light of other generalized form of open sets. By these motivations, we present the concept of nano M-open sets [9] and study their properties and applications in $\mathfrak{N}$ts. The purpose of this paper is to discuss the concept of nano minimal $\mathcal{M}$ open and nano maximal $\mathcal{M}$ open sets and study their properties and applications in $\mathfrak{N}$ts.

Definition 1.1 [13] Let $(U, \tau_R(P))$ be a $\mathfrak{N}$ts and let $A \subseteq U$ then the nano θ-interior (resp. nano θ-closure) of A is defined and denoted by $\mathfrak{N}int_\theta(A) = \cup \{B: B is a No set and \mathfrak{N}cl(B) \subseteq A\}$ (resp. $\mathfrak{N}cl_\theta(A) = \cup \{x \in U: \mathfrak{N}cl(B) \cap A \neq \phi, B is a \mathfrak{N}o set and x \in B\}$).

Definition 1.2 [13] A subset A of P is said to be nano θ-open (resp. nano θ-closed) (briefly, $\mathfrak{N}\theta$o (resp. $\mathfrak{N}\theta$c)) set if $A = \mathfrak{N}int_\theta(A)$ (resp. A^c is a nano θ-open set).

Definition 1.3 [8, 12] Let $(U,\tau_R(P))$ be a $\mathfrak{N}$ts and $A \subseteq U$. Then A is said to be nano regular open (briefly, $\mathfrak{N}$ro) if $A = \mathfrak{N}\mathrm{int}(\mathfrak{N}\mathrm{cl}(A))$.

Definition 1.4 [10] Let $(U,\tau_R(P))$ be a $\mathfrak{N}$ts and let $A \subseteq U$ then the nano δ-interior (resp. nano δ-closure) of A is defined and denoted by $\mathfrak{N}\mathrm{int}_\delta(A) = \bigcup \{B: B is a N ro set and B \subseteq A\}$ (resp. $\mathfrak{N}\mathrm{cl}_\delta(A) = \bigcup \{x \in U: \mathfrak{N}\mathrm{int}(\mathfrak{N}\mathrm{cl}(B)) \cap A \neq \phi,\ B is a \mathfrak{N}o set and x \in B\}$).

Definition 1.5 [10] A subset A of P is said to be nano δ-open (resp. nano δ-closed) (briefly, $\mathfrak{N}\delta o$ (resp. $\mathfrak{N}\delta c$)) set if $A = \mathfrak{N}\mathrm{int}_\delta(A)$ (resp. A^c is a nano δ-open set).

Definition 1.6 Let $(U,\tau_R(P))$ be a $\mathfrak{N}$ts and $A \subseteq U$. Then A is said to be a nano M-open [9](resp. nano M-closed) set (briefly $\mathfrak{N}\mathcal{M}o$ (resp. $\mathfrak{N}\mathcal{M}c$)) if $A \subseteq \mathfrak{N}\mathrm{cl}(\mathfrak{N}\mathrm{int}_\theta(A) \cup \mathfrak{N}\mathrm{int}(\mathfrak{N}\mathrm{cl}_\delta(A))$ (resp. $A \supseteq \mathfrak{N}\mathrm{int}(\mathfrak{N}\mathrm{cl}_\theta(A)) \cap \mathfrak{N}\mathrm{cl}(\mathfrak{N}\mathrm{int}_\delta(A))$).

The family of all $\mathfrak{N}\mathcal{M}o$ (resp. $\mathfrak{N}\mathcal{M}c$) sets are denoted by $\mathfrak{N}\mathcal{M}O(U,\tau_R(P))$, (resp. $\mathfrak{N}\mathcal{M}C(U,\tau_R(P))$).

Definition 1.7 Let $(U,\tau_R(P))$ be a $\mathfrak{N}$ts and let $A \subseteq U$ then the nano M-interior[9] of A is the union of all $\mathfrak{N}\mathcal{M}o$ sets contained in A and denoted by $\mathfrak{N}\mathcal{M}\mathrm{int}(A)$.

Definition 1.8 Let $(U,\tau_R(P))$ be a $\mathfrak{N}$ts and let $A \subseteq U$ then the nano M-closure [9] of A is the intersection of all $\mathfrak{N}\mathcal{M}c$ sets containing A and denoted by $\mathfrak{N}\mathcal{M}\mathrm{cl}(A)$.

Throughout this paper, $(U,\tau_R(P))$ is a $\mathfrak{N}$ts with respect to P where $P \subseteq U$, R is an equivalence relation on U. Then U/R denotes the family of equivalence classes of U by R. All other undefined notions from [7, 8, 11].

2 Nano minimal $\mathcal{M}$ open sets

A non-empty subset A of a $\mathfrak{N}$ts, $(U,\tau_R(P))$ is said to be Nano minimal $\mathcal{M}$ open set (resp. Nano minimal $\mathcal{M}$ closed) (briefly, $\mathfrak{N}\mathrm{Mi}\mathcal{M}o$ (resp. $\mathfrak{N}\mathrm{Mi}\mathcal{M}c$)) if any $\mathfrak{N}\mathcal{M}o$ (resp. $\mathfrak{N}\mathcal{M}c$) set which is contained in A is ϕ or A.

The family of all $\mathfrak{N}\mathrm{Mi}\mathcal{M}o$ (resp. $\mathfrak{N}\mathrm{Mi}\mathcal{M}c$) sets will be denoted by $\mathfrak{N}\mathrm{Mi}\mathcal{M}O(P)$ (resp. $\mathfrak{N}\mathrm{Mi}\mathcal{M}C(P)$). We set $\mathfrak{N}\mathrm{Mi}\mathcal{M}O(P,x) = \{F | x \in F \in \mathrm{Mi}\mathcal{M}O(P)\}$ (resp. $\mathfrak{N}\mathrm{Mi}\mathcal{M}C(P,x) = \{F | x \in F \in \mathrm{Mi}\mathcal{M}C(P)\}$).

Example 2.1 Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{d\},\{b,c\}\}$ and $P = \{b,d\}$. Then the $\mathfrak{N}t\tau_R(P) = \{U,\phi,\{d\},\{b,c\},\{b,c,d\}\}$ which are $\mathfrak{N}o$ sets; the $\mathfrak{N}\mathrm{Mio}$ sets are $\{\{d\},\{b,c\}\}$; the $\mathfrak{N}\mathrm{Mo}$ sets are $\{U,\phi,\{b\},\{c\},\{d\},\{b,c\},\{a,d\},\{b,d\},\{c,d\},\{a,b,d\},\{a,b,c\},\{b,c,d\},\{a,c,d\}\}$; $\mathfrak{N}\mathrm{Mi}\mathcal{M}o$ sets are $\{b\},\{c\},\{d\}$.

The following example shows that $\mathfrak{N}\mathrm{Mio}$ sets and $\mathfrak{N}\mathrm{Mi}\mathcal{M}o$ sets are in general independent.

Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{d\},\{b,c\}\}$ and $P = \{b,d\}$. Then the $\mathfrak{N}t\tau_R(P) = \{U,\phi,\{d\},\{b,c\},\{b,c,d\}\}$ The nano set $\{b\}$ is $\mathfrak{N}\mathrm{Mi}\mathcal{M}o$ which is not $\mathfrak{N}\mathrm{Mio}$ and the set $\{b,c\}$ is $\mathfrak{N}\mathrm{Mio}$ which is not $\mathfrak{N}\mathrm{Mi}\mathcal{M}o$.

Theorem 2.1 For any $\mathfrak{N}$ts, $(U,\tau_R(P))$, [(i)]

1. Let$A$ be a $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{o}$ (resp. $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{c}$) and B be an $\mathfrak{N}\mathcal{M}\mathrm{o}$ (resp. $\mathfrak{N}\mathcal{M}\mathrm{c}$) sets. Then $A \cap B = \phi$ or $A \subset B$.
2. Let A and B be $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{o}$ (resp. $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{c}$) sets. Then $A \cap B = \phi$ or $A = B$.

Proof. (i) Suppose $A \cap B \neq \phi$. Since A is a $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{o}$ (resp. $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{c}$), B be an $\mathfrak{N}\mathcal{M}\mathrm{o}$ (resp. $\mathfrak{N}\mathcal{M}\mathrm{c}$) sets and $A \cap B \subset A$, we have $A \cap B = A$. Therefore $A \subset B$, which implies that every $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{o}$ (resp. $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{c}$) set is a $\mathfrak{N}\mathcal{M}\mathrm{o}$ (resp. $\mathfrak{N}\mathcal{M}\mathrm{c}$) set.

(ii) If $A \cap B \neq \phi$, then $A \subset B$ and $B \subset A$ by (i). Therefore $A = B$.

Theorem 2.2 Let A be a $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{c}$ set in $\mathfrak{N}\mathrm{ts}$, $(U, \tau_R(P))$. [(i)]

1. If $x \in A$ then $A \subset B$ for any $\mathfrak{N}\mathcal{M}\mathrm{c}$ set B containing x.
2. $A = \cap \{B | x \in B \in \mathfrak{N}\mathcal{M}C(P)\}$ for any element x of A.

Proof. (i) Let $A \in \mathfrak{N}\mathrm{Mi}\mathcal{M}C(P, x)$ and $B \in \mathfrak{N}\mathcal{M}C(P, x)$ such that $A \not\subset B$. This implies that $A \cap B \subset A$ and $A \cap B \neq \phi$. But since A is $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{c}$, by (i) of Theorem 2$A \cap B = A$ which contradicts $A \cap B \subset A$. Therefore $A \subset B$.

(ii) By (i) and the fact that A is $\mathfrak{N}\mathcal{M}\mathrm{c}$ set containing x, we have $A \subset \cap \{B | B \in \mathfrak{N}\mathcal{M}C(P, x)\} \subset A$.

Theorem 2.3 If A is a nonempty finite $\mathfrak{N}\mathcal{M}\mathrm{c}$ set of U, then there exists at least one (finite) $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{c}$ set B such that $B \subset A$.

Proof. If A is a $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{c}$ set, we may set $A = B$. If A is not a $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{c}$, then there exists a (finite) $\mathfrak{N}\mathcal{M}\mathrm{c}$ set A_1 such that $\phi \neq A_1 \subset A$. If A_1 is a $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{c}$ set, we may set $B = A_1$. If A_1 is not a $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{c}$, then there exists a (finite) $\mathfrak{N}\mathcal{M}\mathrm{c}$ set A_2 such that $\phi \neq A_2 \subset A_1$. Continuing this process, we have a sequence of $\mathfrak{N}\mathcal{M}\mathrm{c}$ sets.

$$\cdots \subset A_k \subset \cdots A_3 \subset A_2 \subset A_1 \subset A.$$

Since A is a finite set, this process repeats only finitely many times and finally we get a $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{c}$ set $B = A_n$ for some positive integer n.

Theorem 2.4 Let A and $A_\lambda (\lambda \in \Lambda)$ be a $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{c}$ sets in $\mathfrak{N}\mathrm{ts}$, $(U, \tau_R(P))$. [(i)]

1. If $A \subset \bigcup_{\lambda \in \Lambda} A_\lambda$ then then there exists $\lambda \in \Lambda$ such that $A = A_\lambda$.
2. If $A \neq A_\lambda$ for any $\lambda \in \Lambda$, then $(\bigcup_{\lambda \in \Lambda} A_\lambda) \cap A = \phi$.

Proof. (i) It is enough to prove that $A \cap A_\lambda \neq \phi$. Suppose $A \cap A_\lambda = \phi$. then $A_\lambda \subset U \backslash A$ and hence $A \subset \bigcup_{\lambda \in \Lambda} A_\lambda \subset U \backslash A$ which is a contradiction. Now as $A \cap A_\lambda \neq \phi$, then $A \cap A_\lambda \subset A$ and $A \cap A_\lambda \subset A_\lambda$. Since $A \cap A_\lambda \subset A$ and given that A is $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{c}$, then by definition $A \cap A_\lambda = A$ or $A \cap A_\lambda = \phi$. But $A \cap A_\lambda \neq \phi$. Then $A \cap A_\lambda = A$ which implies $A \subset A_\lambda$. Similarly if $A \cap A_\lambda \subset A_\lambda$ and given that A_λ is $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{c}$, then by definition $A \cap A_\lambda = A_\lambda$ or $A \cap A_\lambda = \phi$. But $A \cap A_\lambda \neq \phi$ then $A \cap A_\lambda = A_\lambda$ which implies $A_\lambda \subset A$. Then $A = A_\lambda$.

(2) Suppose that $(\bigcup_{\lambda \in \Lambda} A_\lambda) \cap A \neq \phi$. Then there exists $\lambda \in \Lambda$ such that $A_\lambda \cap A \neq \phi$. By (ii) of Theorem 2, we have $A = A_\lambda$ which contradicts the fact that $A \notin A_\lambda$ for any $\lambda \in \Lambda$. Hence $(\bigcup_{\lambda \in \Lambda} A_\lambda) \cap A = \phi$

3 Nano Maximal $\mathcal{M}$-open sets

Definition 3.1 A proper nonempty $\mathfrak{N}\mathcal{M}\mathrm{o}$ set A of U is called a nano maximal $\mathcal{M}$-open (briefly, $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$) set if any $\mathfrak{N}\mathcal{M}\mathrm{o}$ set which contains A is either U or A.

The family of all $\mathfrak{N}\mathrm{Ma}\mathcal{M}$-open sets will be denoted by $\mathfrak{N}\mathrm{Ma}\mathcal{M}O(P)$. We set $\mathfrak{N}\mathrm{Ma}\mathcal{M}O(P, x) = \{A | x \in A \in \mathfrak{N}\mathrm{Ma}\mathcal{M}O(P)\}$.

Example 3.1 Let $U = \{a, b, c, d, e\}$ with $\frac{U}{R} = \{\{c\}, \{a, b\}, \{d, e\}\}$ and $P = \{a, c\}$. Then the $\mathfrak{N}\mathrm{t}\tau_R(P) = \{U, \phi, \{c\}, \{a, b\}, \{a, b, c\}\}$ which are $\mathfrak{N}\mathrm{o}$ sets; the $\mathfrak{N}\mathrm{Mao}$ sets are $\{\{a, b, c\}\}$; the $\mathfrak{N}\mathcal{M}\mathrm{o}$ sets are

Vol 10, Issue 11, NOV/2019
ISSN NO:0377-9254

$\{U, \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{a,c,d\}, \{a,c,e\}, \{b,c,d\}, \{b,c,e\}, \{c,d,e\}, \{a,b,c,d\}, \{a,b,c,e\}, \{a,b,d,e\}, \{a,c,d,e\}, \{b,c,d,e\}\}$;$\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ sets are $\{a,b,c,d\}, \{a,b,c,e\}, \{a,b,d,e\}, \{a,c,d,e\}, \{b,c,d,e\}$.

The following example shows that $\mathfrak{N}\mathrm{Mao}$ sets and $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{o}$ sets are in general independent.

Example 3.2 In Example 3, the nano set $\{a,b,c\}$ is $\mathfrak{N}\mathrm{Mao}$ which is not $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ and the set $\{a,b,c,d\}$ is $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ which is not $\mathfrak{N}\mathrm{Mao}$.

Theorem 3.1 Let A be a proper nonempty subset A of U. Then A is $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ if and only if $U\backslash A$ is a $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{c}$ set.

Proof. Necessity. Let A be a $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ set. Then $A \subset U$ or $A \subset A$. Hence $\phi \subset U\backslash A$ or $U\backslash A \subset U\backslash A$. Therefore by Definition 3, $U\backslash A$ is a $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{c}$ set.

Sufficiency. Let $U\backslash A$ is a $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{c}$ set. Then $\phi \subset U\backslash A$ or $U\backslash A \subset U\backslash A$. Hence $A \subset U$ or $A \subset A$ which implies that A is a $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ set.

Theorem 3.2 Let A be a $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ set and if B be a [(i)]

1. $\mathfrak{N}\mathcal{M}\mathrm{o}$ set. Then $A \cup B = U$ or $B \subset A$.
2. $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ set. Then $A \cup B = U$ or $B = A$.

Proof. (i) If $A \cup B = U$ then we are done. But if $A \cup B \neq U$, then we have to prove that $B \subset A$. Now $A \cup B \neq U$ means $B \subset A \cup B$ and $A \subset A \cup B$. Therefore we have $A \subset A \cup B$ and A is $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$, then by definition $A \cup B = U$ or $A \cup B = A$ but $A \cup B \neq U$, then $A \cup B = A$ which implies $B \subset A$.

(ii) If $A \cup B = U$, then we are done. But if $A \cup B \neq U$, then we have to prove that $B = A$. Now $A \cup B \neq U$ means $A \subset A \cup B$ and $B \subset A \cup B$. Now $A \subset A \cup B$ and A is $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ set, then by definition $A \cup B = U$ or $A \cup B = A$ but $A \cup B \neq U$. Therefore $A \cup B = A$ which implies $B \subset A$. Similarly if $B \subset A \cup B$, we obtain $A \subset B$. Therefore $A = B$.

Theorem 3.3 Let A be a $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ [(i)]

1. If x be an element of $U\backslash A$, then $U\backslash A \subset B$ for any $\mathfrak{N}\mathcal{M}\mathrm{o}$ set B containing x.
2. either (a) or (b) holds [(a)]
   (a) For each $x \in U\backslash A$ and each $\mathfrak{N}\mathcal{M}\mathrm{o}$ set B containing $x, B = U$.
   (b) There exists a $\mathfrak{N}\mathcal{M}\mathrm{o}$ set B such that $U\backslash A \subset B$ and $B \subset U$.
3. either (a) or (b) holds [(a)]
4. For each $x \in U\backslash A$ and each $\mathfrak{N}\mathcal{M}\mathrm{o}$ set B containing x we have $U\backslash A \subset B$.
5. There exists a $\mathfrak{N}\mathcal{M}\mathrm{o}$ set B such that $U\backslash A = B \neq U$.

Proof. (i) Since $x \in U\backslash A$, we have $B \not\subset A$ for any $\mathfrak{N}\mathcal{M}\mathrm{o}$ set B containing x. Then $A \cup B = U$ by (i) of Theorem 2 Therefore $U\backslash A \subset B$.

(ii) If (a) does not hold, then there exist an element x of $U\backslash A$ and a $\mathfrak{N}\mathcal{M}\mathrm{o}$ set B containing x such that $B \subset U$. By (i) we have $U\backslash A \subset B$.

(3) If (b) does not hold, then, by (i), we have $U\backslash A \subset B$ for each $x \in U\backslash A$ and each $\mathfrak{N}\mathcal{M}\mathrm{o}$ set B containing x. Hence $U\backslash A \subset B$.

Theorem 3.4 Let A, B, C be $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ sets such that $A \neq B$. If $A \cap B \subset C$, then either $A = C$ or $B = C$.

Proof. Given that $A \cap B \subset C$. If $A = C$ then there is nothing to prove. But if $A \neq C$, then we have to prove $B = C$. By using (ii) of Theorem 2 we have : $B \cap C = B \cap [C \cap X] = B \cap [C \cap (A \cup B)] = B \cap [(C \cap A) \cup (C \cap B)] = (B \cap C \cap A) \cup (B \cap C \cap B) = (A \cap B) \cup (C \cap B) (\text{since} A \cap B \subset C) = (A \cup C) \cap B = U \cap B = B, (\text{since} A \cup C = U)$. This implies $B \subset C$. It follows from the definition of

$\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ set that $B = C$

Theorem 3.5 Let A, B, C be $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ sets which are different from each other. Then $(A \cap B) \not\subset (A \cap C)$.

Proof. Let $(A \cap B) \subset (A \cap C)$. Then, $(A \cap B) \cup (C \cap B) \subset (A \cap C) \cup (C \cap B)$. Hence $(A \cup C) \cap B \subset C \cap (A \cup B)$. Since by (ii) of Theorem $2A \cup C = U$ and $A \cup B = U$, we have $U \cap B \subset C \cap U$ which implies $B \subset C$. From definition of $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ set it follows that $B = C$. This contradicts the fact that A, B and C are different from each other. Therefore $(A \cap B) \not\subset (A \cap C)$.

We call a set cofinite if its complement is finite.

Theorem 3.6 If A is a proper nonempty cofinite $\mathfrak{N}\mathcal{M}\mathrm{o}$ set of U, then there exists (cofinite) $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ set B such that $A \subset B$.

Proof. If A is a $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ set, we may set $A = B$. If A is not a $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$, then there exists a (cofinite) $\mathfrak{N}\mathcal{M}\mathrm{o}$ set A_1 such that $A \subset A_1 \neq U$. If A_1 is a $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ set, we may set $B = A_1$. If A_1 is not a $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ set, then there exists a (cofinite) $\mathfrak{N}\mathcal{M}\mathrm{o}$ set $A_2 \neq U$ such that $A \subset A_1 \subset A_2 (\neq U)$. Continuing this process, we have a sequence of $\mathfrak{N}\mathcal{M}\mathrm{o}$ sets such that $A \subset A_1 \subset A_2 \subset \cdots A_k \subset \cdots$ Since A is a cofinite set, this process repeats only finitely many times and finally we get a $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ set $B = A_n$ for some positive integer n

Theorem 3.7 The following statements are true for any topological space U.

(1) Let A be a $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ set of U. Then either $\mathfrak{N}\mathcal{M}\mathrm{cl}(A) = U$ or $\mathfrak{N}\mathcal{M}\mathrm{cl}(A) = A$.

(2) Let A be a $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ set of U. Then either $\mathfrak{N}\mathcal{M}\mathrm{int}(U\backslash A) = U\backslash A$ or $\mathfrak{N}\mathcal{M}\mathrm{int}(U\backslash A) = \phi$.

Proof. (1) Since A is a $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ set, only the following cases (i) and (ii) are possible by Theorem 3.4(3). (i) Let $\mathfrak{N}\mathcal{M}\mathrm{cl}(A) \neq U$. Then there exists $x \in U\backslash\mathfrak{N}\mathcal{M}\mathrm{cl}(A)$. Hence there exists a $\mathfrak{N}\mathcal{M}\mathrm{o}$ set B such that $x \in B$ and $B \cap A = \phi$. Therefore, $B \subset U\backslash A$. On the other hand, by Theorem 3.4(1), $U\backslash A \subset B$ and $B = U\backslash A$. Hence A is $\mathfrak{N}\mathcal{M}\mathrm{c}$ and $\mathfrak{N}\mathcal{M}\mathrm{cl}(A) = A$.

(2) By (1), we have $\mathfrak{N}\mathcal{M}\mathrm{cl}(A) = A$ or $\mathfrak{N}\mathcal{M}\mathrm{cl}(A) = U$. Hence $\mathfrak{N}\mathcal{M}\mathrm{int}(U\backslash A) = U\backslash A$ or $\mathfrak{N}\mathcal{M}\mathrm{int}(U\backslash A) = \phi$.

Theorem 3.8 The following statements are true for any topological space U. (1) Let A be a $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ set of U and B a nonempty subset of $U\backslash A$. Then $\mathfrak{N}\mathcal{M}\mathrm{cl}(B) = U\backslash A$.

(2) Let A be a $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ set of U. and G a proper subset of U with $A \subset G$. Then $\mathfrak{N}\mathcal{M}\mathrm{int}(G) = A$.

Proof. (1) Since $\phi \neq B \subset U\backslash A$, by Theorem 3.4(1) we have that $W \cap A \neq \phi$ for $x \in U\backslash A$ and $\mathfrak{N}\mathcal{M}\mathrm{o}$ set W containing x. Hence $W \cap B \neq \phi$ for any $\mathfrak{N}\mathcal{M}\mathrm{o}$ set W containing x. Thus, $U\backslash A \subset \mathfrak{N}\mathcal{M}\mathrm{cl}(B)$. Since $U\backslash A$ is $\mathfrak{N}\mathcal{M}\mathrm{c}$ and $B \subset U\backslash A$, we have $\mathfrak{N}\mathcal{M}\mathrm{cl}(B)\mathrm{subset}U\backslash A$. Therefore $\mathfrak{N}\mathcal{M}\mathrm{cl}(B) = U\backslash A$.

(2) If $G = A$, then $\mathfrak{N}\mathcal{M}\mathrm{int}(G) = \mathfrak{N}\mathcal{M}\mathrm{int}(A) = A$. If $G \neq A$, then we have $A \subset G$. Thus $A \subset \mathfrak{N}\mathcal{M}\mathrm{int}(G)$. Since A is $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ and $\mathfrak{N}\mathcal{M}\mathrm{int}(G)$ is $\mathfrak{N}\mathcal{M}\mathrm{o}$ containing A, then, $\mathfrak{N}\mathcal{M}\mathrm{int}(G) = U$ or $\mathfrak{N}\mathcal{M}\mathrm{int}(G) = A$. Since G is proper, $\mathfrak{N}\mathcal{M}\mathrm{int}(G) \neq U$. Therefore $\mathfrak{N}\mathcal{M}\mathrm{int}(G) = A$.

4 Conclusion

The authors study $\mathfrak{N}\mathrm{Mi}\mathcal{M}\mathrm{o}$ and $\mathfrak{N}\mathrm{Ma}\mathcal{M}\mathrm{o}$ sets in $\mathfrak{N}$ts's. Properties and characterizations of these sets are investigated.

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