

# Large Size ECG Signal Effective Reconstruction and Compression by Tchebichef

<sup>1</sup>Sruthi Dadi

Assistant Professor-Department of ECE,  
Annamacharya Institute of Technology & Sciences

**Abstract**—In this paper, we propose an efficient method for the compression and reconstruction of the large-size 1D electrocardiogram (ECG) signal. In this method, we use Tchebichef moments calculated by the modified Gram-Schmidt ortho-normalization process with the use of a full large-sized signal (N=8000) in the reconstruction and compression process. The simulation results and the comparisons carried out show the efficiency and the superiority of the proposed compression and reconstruction methods in terms of the high quality of the compressed signals and in terms of the high compression ratio.

**Keywords**—1D signal analysis, Tchebichef polynomials, Tchebichef moments, ECG signal compression, ECG signal reconstruction, discrete orthogonal polynomials.

## I. INTRODUCTION

Discrete orthogonal moments (DOMs) are efficient tools since they are widely used in the field of signal and image analysis. Applications of DOMs include pattern recognition, contour detection, watermarking, reconstruction and compression. The computation of DOMs is based on discrete orthogonal polynomials (DOPs) such as de Krawchouk [1], Hahn [2], Meixner [3], [4], Charlier [5], [6], [7] dual Hahn [8], Racah [9]. The latter are defined from the very complicated hypergeometric functions that cause the overflow problem (Nan and Inf) during their implementation. To overcome this problem, the researchers calculate DOPs using ortho-normalized forms and three-term recurrence relationships with respect to the order  $n$  or with respect to the variable  $x$ . However, the problem of appearance and propagation of numerical errors due to the implementation under machine appears. This problem worsens the computation results and causes the destruction of the orthogonality property of the DOPs. To overcome this problem, Camacho-bello et al. [10] and Daoui et al. [2], [5] use the modified Gram-Schmidt ortho-normalization process (MGSOP). The use of the latter limits the propagation of numerical errors in recursive computation. Consequently, MGSOP maintains the numerical stability of high-order DOPs. Among DOPs, we find that Tchebichef polynomials (TPs) occupy a special

position especially in reconstruction and compression applications. This peculiarity is due to the fact that TPs possess an important energy compacting property even for the lower polynomial orders

[11]. As a result, a high reconstruction quality and high compression ratio can be achieved. For these reasons, we use in this paper TPs and Tchebichef moments (TMs) for the reconstruction and lossy compression of large-sized electrocardiogram (ECG) signal.

The transformation-based lossy compression algorithms have been widely applied for image and signal compression. One of the most widely used algorithms for lossy compression of ECG signals is the Discrete Cosine Transform (DCT), due to its simplicity of implementation and good compression results [12], [13]. Discrete Wavelet Transformations (DWT) also make an interesting techniques for the compression of ECG signals [14], [15].

Recently, discrete orthogonal moments (transformations) have received increasing attention in signal and image analysis, since they can efficiently represent information without information redundancy [16]. Among the different types of moments, we find that Tchebichef moments have been widely and successfully applied for image and signal

compression due to their less computational complexity, and they provide high quality of the compressed signals and images [17]–[20]. It can be concluded that most of the works concerning the compression using Tchebichef moments are based on the block representation of the signal/image. This representation has been applied for large-sized signals/images in order to avoid the numerical instability and to maintain an efficient reconstruction by Tchebichef moments [19]. However, compression based on small or medium block (  $8 \times 8$  ,  $16 \times 16$  ,  $32 \times 32$  ) representation does not achieve a high compression ratio [17]. Motivated by the need to achieve a very high compression ratio while preserving a good quality of the reconstructed signal, we propose in this paper an efficient method for the compression of large-sized ECG signals that require large storage space. Therefore, the main contribution of this paper is to propose an efficient method for the compression of large-sized ECG signals. This proposed method is essentially based on: (i) the use of full signal in the compression process instead of block representation in order to achieve a very high compression ratio. (ii) the use of modified Gram-Schmidt orthonormalization procedures to maintain the numerical stability of Tchebichef moments during the compression of large-sized ECG signals. In addition, the proposed method is simple, stable for signals of any size, fast to implement and does not require any pre-processing or post-processing of the compressed signal.

The rest of this document is organized as follows. Section 2 presents a brief overview on Tchebichef polynomials and moments. The proposed method of large-sized ECG signal compression is presented in section 3. The numerical simulation results are discussed in section 4. Finally, section 5 concludes this paper.

## II. DISCRETE ORTHOGONAL TCHEBICHEF POLYNOMIALS AND MOMENTS

In this section, we briefly present the theoretical background of TPs and TMs

### A. Recursive Computation of TPs with Respect to the Order n

The orthonormalized Tchebichef polynomials with respect to the order n are defined by the following relation [21]:

$$\tilde{t}_n(x) = \frac{t_n(x)}{\sqrt{\rho(n, N)}} \quad (1)$$

with  $t_n(x)$  are the Tchebichef polynomials defined by :

$$t_n(x) = n! \sum_{k=0}^n (-1)^{n-k} \binom{N-1-k}{n-k} \binom{n+k}{n} \binom{x}{k} \quad (2)$$

and  $\rho(n, N)$  is the square norm function defined as :

$$\rho(n, N) = (2n)! \binom{N+n}{2n+1} \quad (3)$$

The direct computation of the TPs from Eq (1) is time consuming and generates the overflow of the polynomial values due to the factorial function. Noting that under Matlab, we find  $170! = 7.2574 \times 10306$  and  $180! = \text{Inf}$  because the largest floating-point number that can be represented using Matlab system is  $1.797693134862316 \times 10308$  [22]. To overcome this problem, Mukundan [21] developed the recursive calculation of the normalized Tchebichef polynomials. Indeed, the three-term recurrence relation of these polynomials with respect to the order n, is given by [21]:

$$\begin{aligned} \tilde{t}_n(x) &= \frac{\alpha x + \beta}{n} \tilde{t}_{n-1}(x) + \frac{\mu}{n} \tilde{t}_{n-2}(x) \\ \alpha &= \frac{2}{n} \sqrt{\frac{4n^2-1}{N^2-n^2}}, \quad \beta = \frac{1-N}{n} \sqrt{\frac{4n^2-1}{N^2-n^2}} \\ \mu &= \frac{1-n}{n} \sqrt{\frac{2n+1}{2n-3}} \sqrt{\frac{N^2-(n-1)^2}{N^2-n^2}} \end{aligned} \quad (4)$$

with  $n \times N = -2, 3, \dots, 1$  and  $0, 1, \dots, 1$ , and the initial conditions of TPs are given by :

$$\tilde{t}_0(x) = \frac{1}{\sqrt{N}} \quad \text{and} \quad \tilde{t}_1(x) = \frac{2x+1-N}{N} \sqrt{\frac{3}{N(N^2-1)}} \quad (5)$$

However, in recursive computation, the problem of numerical error accumulation and propagation appears. This problem destroys the orthogonality property of TPs.

To resolve this problem, we find that Camacho-bello et al. and Daoui et al. [2], [5] use the modified Gram-Schmidt ortho-normalization process (MGSOP). Indeed, the implementation of TPs using MGSOP is performed by the following algorithm:

**ALGORITHM 1: THE COMPUTATION OF TPs WITH RESPECT TO THE ORDER n USING MGSOP**

```

Inputs: N: The maximum value of the variable x, N_max :
TPs order
Outputs :MPs matrix
Step1 : Compute  $\tilde{t}_0(x)$  and  $\tilde{t}_1(x)$  using Eq.(5)
Step 2: Compute  $\tilde{t}_n(x)$  using Eq.(4)
Step 3: Ortho-normalized TPs values by GSOP as follows
x = 0 : N - 1
for n ← 1 to N_max do
GS_n(x) =  $\tilde{t}_n(x)$ 
for j ← 0 to n - 1 do
GS_n(x) = GS_n(x) - GS_j(x)^T GS_n(x) GS_j(x)
end for
E_n(x) = GS_n(x) / ||GS_n(x)||_2
 $\tilde{t}_n(x) = E_n(x)$ 

```

The use of Algorithm 1 allows to obtain the results shown in Fig.1.

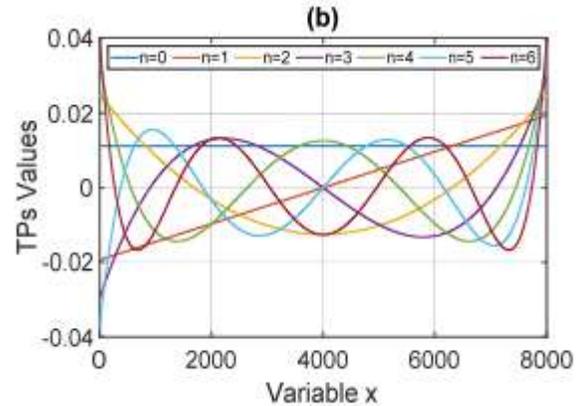
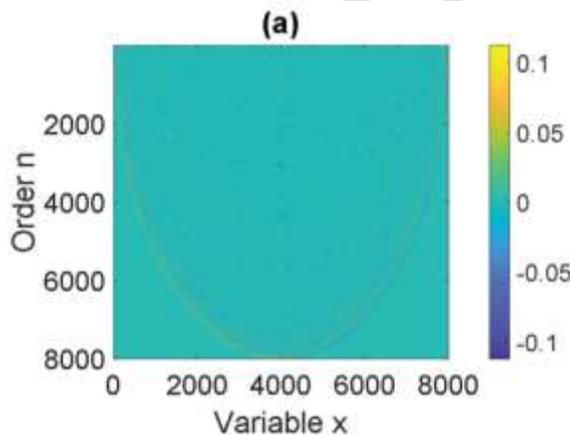


Fig. 1. (a): TPs matrix for n, x = 0 : 8000 , (b) : TPs up to the sixth order.

The results obtained (Fig.1) clearly show that the calculation of TPs is numerically stable for high TPs orders (n=8000). This indicates that the use of MGSOP overcomes the problem of numerical error propagation during the computation of TPs.

**B. Tchebichef Moments for Signal Reconstruction**

The TMs for a one-dimensional signal are defined by the following matrix formulation :

$$TM_n = T_n^T f \text{ with } T_n = [\tilde{t}_0(x), \tilde{t}_1(x), \dots, \tilde{t}_n(x)]^T, \text{ with } x = 0 : N - 1 \text{ and } n \leq N \tag{6}$$

where  $f = f(x)$  with  $x = 1, 2, \dots, N$  denotes the signal vector of length  $1 \times N$ .

The inverse operation (reconstruction) of a 1D signal is given by the following relation :

$$\hat{f} = TM_n T_n^T \tag{7}$$

**III. PROPOSED METHOD OF LARGE SIZE SIGNAL COMPRESSION BY TCHEBICHEF MOMENTS**

Methods of compressing a large-sized signal  $N > 1000$  using TPs involves subdividing the signal into smaller blocks of sizes  $1 \times nb$  with  $nb \ll N$  (e.g.  $1 \times 8$ ,  $1 \times 16$  or  $1 \times 32$ ). This method is mainly used to avoid the digital instability of high order TPs [17]. Since the instability of TPs is solved by the use of MGSOP, we propose to use

the full signal size instead of blocks in order to obtain a high compression ratio (CR). Knowing that the high CR is obtained when using blocks of large sizes [17]. The proposed compression method is summarized in the following flowchart:

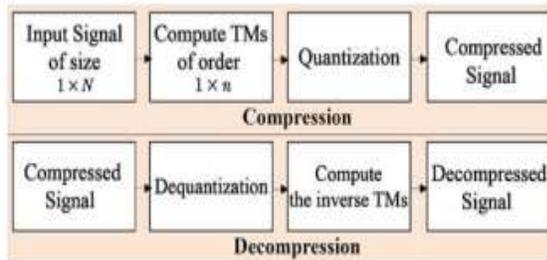


Fig.2 Flowchart of the proposed signal compression method.

The signal compression ratio (CR %) is determined by the following relationship:

$$CR = \frac{\#bits\ to\ represent\ f}{\#bits\ to\ represent\ \hat{f}} \quad (8)$$

where  $f$  is the original signal and  $\hat{f}$  is the compressed signal file. The order of TPs to obtain the desired CR is given by the following relation:

$$n = round\left(\frac{N}{CR} \times \frac{2^i}{2^j}\right) \quad (9)$$

where  $N$  is the signal dimension,  $n$  is the order of TPs,  $i$  and  $j$  are values depending on the class of the original signal data and TMs. The decompressed signal quality is measured by the following criteria : • The percentage root mean-square difference (PRD%) defined by :

$$PRD\% = \sqrt{\frac{\sum_{x=0}^{N-1} (f(x) - \hat{f}(x))^2}{\sum_{x=0}^{N-1} f(x)^2}} \times 100 \quad (10)$$

Mean-Square Error (MSE): the reconstruction error between the original image and the reconstructed image, MSE is defined for a 3D image size by :

$$MSE = \frac{1}{N} \sum_{x=0}^{N-1} (f(x) - \hat{f}(x))^2 \quad (11)$$

with  $f(x)$  is the original signal and  $\hat{f}(x)$  is the reconstructed signal.

#### IV. SIMULATION RESULTS AND DISCUSSIONS

Methods of compressing a large-sized signal  $N > 1000$  using TPs involves subdividing the signal into smaller blocks of sizes  $1 \times nb$  with  $nb \ll N$  (e.g.  $1 \times 8$ ,  $1 \times 16$  or  $1 \times 32$ ). This method is mainly used to avoid the digital instability of high order TPs [17]. Since the instability of TPs is solved by the use of MGSOP, we propose to use the full signal size instead of blocks in order to obtain a high compression ratio.

##### A. Large-size ECG signal reconstruction

In the following test, an ECG-type signal named "MITBIH recorder 111" is used. This signal selected from the database [23] is (calculated by the MGSOP method), Hahn [2], Krawchouk [1] and Charlier [5] moments. Then the quality of the reconstructed signal is compared using the different types of moments. From the results shown in Fig.3, the superiority of TMs over other kinds of moments is clearly shown since they are able to reconstruct the signal with high quality reconstruction (minimum MSE) even for the lower moment orders. Indeed, Fig.4 shows a set of reconstructed signals by TMs for different orders. The resemblance between the reconstructed signal and the original signal is clear, which justifies the superiority of TMs compared to other types of moments.

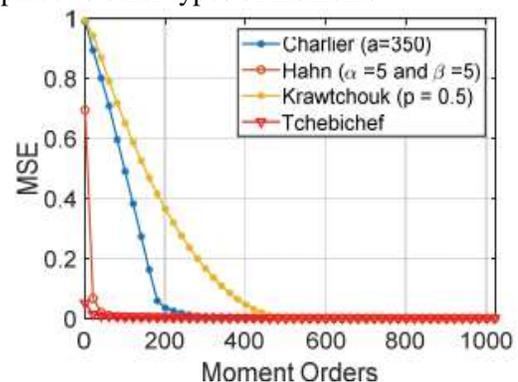


Fig.3. MSE of the reconstructed signal "MITBIH recorder 111" using different types of moments.

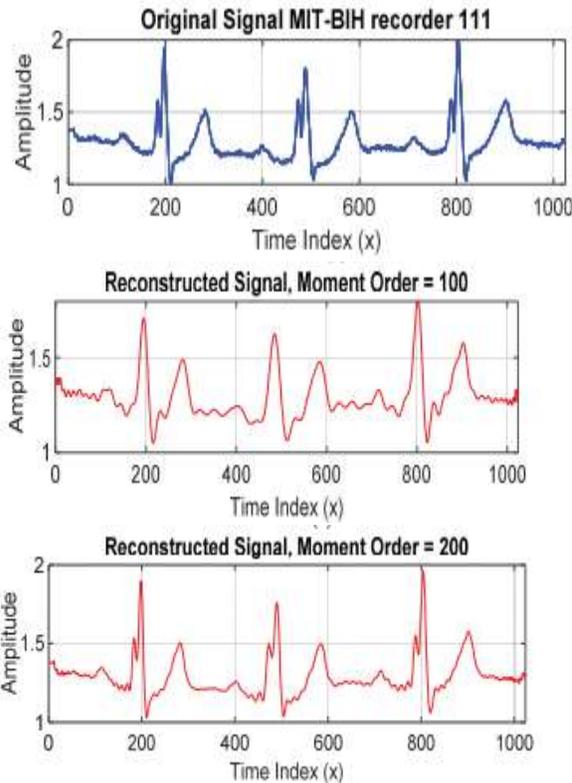


Fig.3. Set of reconstructed signals by TMs of different orders using the "MIT-BIH recorder 111" signal.

**B. Compression of large-size ECG signals**

In this test, we use two recordings of ECG signals referred as "MIT-BIH recorder 101" and "ECG-ID person 02/rec". These signal selected from the database [23] sizes are compressed by the proposed method. The compression results are shown in Figs.4 and 5.

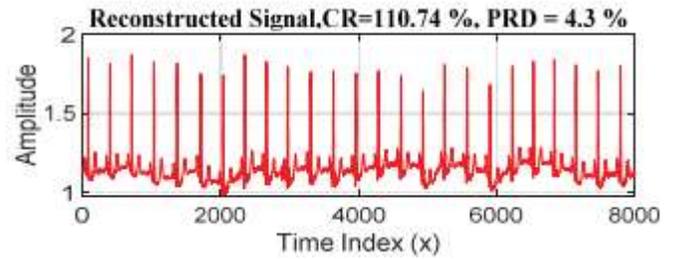
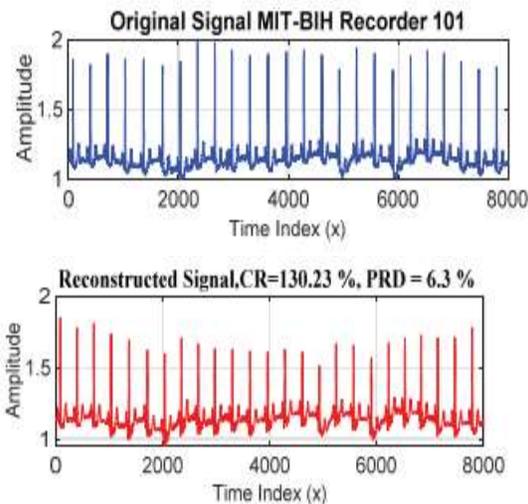


Fig.4. Set of decompressed "MIT-BIH recorder 101" signals.

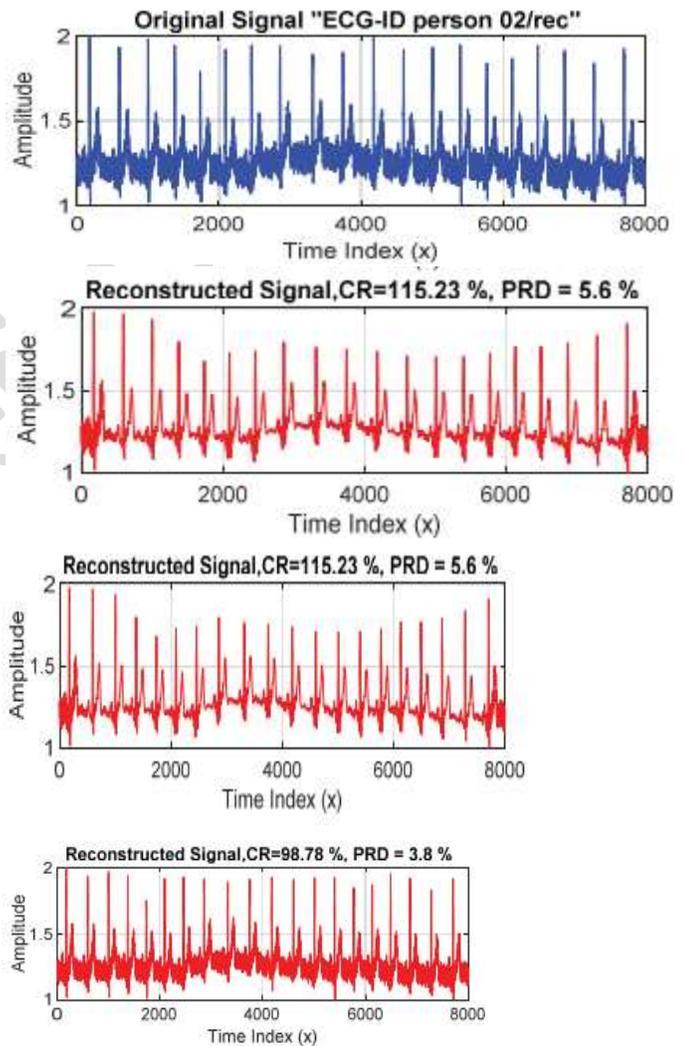


Fig.5. Set of decompressed "ECG-ID person 02/rec" signals.

The obtained results (Figs. 4 and 5) clearly show that the proposed compression method achieves a high CR with minimal reconstruction errors (low PRD% values). These important results validate the usefulness of the proposed compression method for the efficient

compression of largesize signals. To justify the effectiveness of the proposed method, a comparison with other recent methods presented in [15], [17], [24], [25] is presented. In this comparison we quantify the error of the decompressed signal by the PRD% criterion and we measure the compression ratio by the criterion CR%. The results obtained are shown in Fig.6.

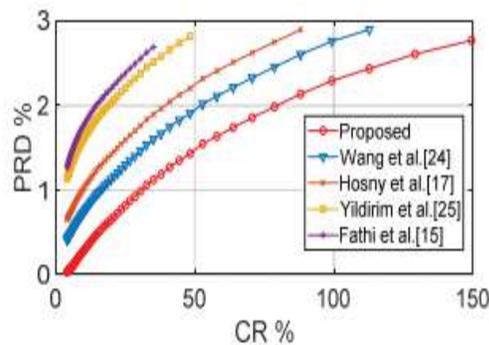


Fig.6. Comparison of the proposed compression method with other recent signal compression methods.

The results obtained (Fig. 6) show that for the same CR%, the proposed method offers a lower PRD% error compared to the other methods. In addition, the suggested method is more efficient than all other methods used in the test in terms of CR% since it achieves a maximum CR (CR = 150%) with a minimum PRD error (PRD=2.86%). Unlike all the other methods used in the test which do not allow to reach such a CR%. These interesting results clearly justify the effectiveness of the proposed compression method for the compression of large-sized ECG signals.

## V. CONCLUSION

In this paper, we used TPs computed by the MGSOP for the reconstruction and the compression of large-sized ECG signals. The efficiency of our proposed compression method is illustrated in terms of high compression ratio and low errors of the compressed signal. The results of the performed simulations have clearly justified the superiority of the proposed compression method over other recent methods for the compression of large-sized ECG signals. The proposed compression method will be used in the future for the compression of large-sized 2D/3D images.

## REFERENCES

- [1] B. Honarvar Shakibaei Asli et J. Flusser, « Fast computation of Krawtchouk moments », *Inf. Sci.*, vol. 288, p. 7386, déc. 2014, doi: 10.1016/j.ins.2014.07.046.
- [2] A. Daoui, M. Yamni, O. E. Ogri, H. Karmouni, M. Sayyouri, et H. Qjidaa, « New Algorithm for Large-Sized 2D and 3D Image Reconstruction using Higher-Order Hahn Moments », *Circuits Syst. Signal Process.*, mars 2020, doi: 10.1007/s00034-020-01384-z.
- [3] H. Karmouni, T. Jahid, A. Hmimid, M. Sayyouri, et H. Qjidaa, « Fast computation of inverse Meixner moments transform using Clenshaw's formula », *Multimed. Tools Appl.*, vol. 78, no 22, p. 3124531265, nov. 2019, doi: 10.1007/s11042-019-07961-y.
- [4] A. Hmimid, M. Sayyouri, et H. Qjidaa, « Image classification using separable invariant moments of Charlier-Meixner and support vector machine », *Multimed. Tools Appl.*, vol. 77, no 18, p. 2360723631, sept. 2018, doi: 10.1007/s11042-018-5623-3.
- [5] A. Daoui, M. Yamni, O. El ogri, H. Karmouni, M. Sayyouri, et H. Qjidaa, « Stable computation of higher order Charlier moments for signal and image reconstruction », *Inf. Sci.*, vol. 521, p. 251276, juin 2020, doi: 10.1016/j.ins.2020.02.019.
- [6] H. Karmouni et al., « Image reconstruction by Krawtchouk moments via digital filter », in *2017 Intelligent Systems and Computer Vision (ISCV)*, 2017p. 1–7.
- [7] A. Hmimid, M. Sayyouri, et H. Qjidaa, « Image classification using novel set of Charlier moment invariants », *WSEAS Trans Signal Process.*, vol. 10, no 1, p. 156–167, 2014.
- [8] H. Zhu, H. Shu, J. Zhou, L. Luo, et J. L. Coatrieux, « Image analysis by discrete orthogonal dual Hahn moments », *Pattern Recognit. Lett.*, vol. 28, no 13, p. 1688, oct. 2007, doi: 10.1016/j.patrec.2007.04.013.
- [9] H. Zhu, H. Shu, J. Liang, L. Luo, et J.-L. Coatrieux, « Image analysis by discrete orthogonal Racah moments », *Signal Process.*, vol. 87, no 4, p. 687708, avr. 2007, doi: 10.1016/j.sigpro.2006.07.007.

- [10] C. Camacho-Bello et J. S. Rivera-Lopez, « Some computational aspects of Tchebichef moments for higher orders », *Pattern Recognit. Lett.*, vol. 112, p.332-339, sept.2018, doi: 10.1016/j.patrec.2018.08.020.
- [11] H. Zhu, M. Liu, H. Shu, H. Zhang, et L. Luo, « General form for obtaining discrete orthogonal moments », *IET Image Process.*, vol. 4, no 5, p. 335-352, oct. 2010, doi: 10.1049/iet-ipr.2009.0195.
- [12] A. Bendifallah, R. Benzid, et M. Boulemden, « Improved ECG compression method using discrete cosine transform », *Electron. Lett.*, vol. 47, no 2, p. 87–89, 2011.
- [13] M. Kiran Kumar, Dr. K. Bhargavi. *An Effective Study on Data Science Approach to Cybercrime Underground Economy Data. Journal of Engineering, Computing and Architecture.2020;p.148.*
- [14] M. Kiran Kumar , S. Jessica Saritha. *AN EFFICIENT APPROACH TO QUERY REFORMULATION IN WEB SEARCH, International Journal of Research in Engineering and Technology. 2015;p.172.*
- [15] M KIRAN KUMAR, K BALAKRISHNA, M NAGA SESHUDU, A SANDEEP. *Providing Privacy for Numeric Range SQL Queries Using Two-Cloud Architecture. International Journal of Scientific Research and Review. 2018;p.39*
- [16] K BALA KRISHNA, M NAGASESHUDU, M KIRAN KUMAR. *An Effective Way of Processing Big Data by Using Hierarchically Distributed Data Matrix. International Journal of Research.2019;p.1628*
- [17] P.Padma, Vadapalli Gopi, M.Kiran Kumar. *Detection of Cyber anomaly Using Fuzzy Neural networks. Journal of Engineering Sciences.2020;p.48.*
- [18] F. Ernawan, N. Kabir, et K. Z. Zamli, « An efficient image compression technique using Tchebichef bit allocation », *Optik*, vol. 148, p. 106-119, nov. 2017, doi: 10.1016/j.ijleo.2017.08.007.
- [19] W. S. Lang, N. A. Abu, et H. Rahmalan, « Fast 4x4 Tchebichef moment image compression », in *2009 International Conference of Soft Computing and Pattern Recognition*, 2009, p. 295–300.
- [20] B. Xiao, G. Lu, Y. Zhang, W. Li, et G. Wang, « Lossless image compression based on integer Discrete Tchebichef Transform », *Neurocomputing*, vol. 214, p. 587–593, nov. 2016, doi: 10.1016/j.neucom.2016.06.050.
- [21] R. Mukundan, S. H. Ong, et P. A. Lee, « Image analysis by Tchebichef moments », *IEEE Trans. Image Process.*, vol. 10, no 9, p. 1357–1364, 2001.
- [22] C. B. Moler, *Numerical Computing with MATLAB*. Society for Industrial and Applied Mathematics, Philadelphia. Crossref, 2004.
- [23] G. B. Moody et R. G. Mark, « The impact of the MIT-BIH arrhythmia database », *IEEE Eng. Med. Biol. Mag.*, vol. 20, no 3, p. 45–50, 2001.
- [24] F. Wang et al., « A novel ECG signal compression method using spindle convolutional auto-encoder », *Comput. Methods Programs Biomed.*, vol. 175, p. 139–150, juill. 2019, doi: 10.1016/j.cmpb.2019.03.019.
- [25] O. Yildirim, R. S. Tan, et U. R. Acharya, « An efficient compression of ECG signals using deep convolutional autoencoders », *Cogn. Syst. Res.*, vol. 52, p. 198-211, déc. 2018, doi: 10.1016/j.cogsys.2018.07.004.