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Lehmer -3 Mean Labeling of Disconnected Graphs Related to K_3

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Abstract

A graph $G=(V,E)$ with p vertices and q edges is called a Lehmer -3 Mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from 1,2,…,q+1 in such a way that when each edge $e=uv$ is labeled with
f($e=uv$)= $\left\lceil\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rceil$ (or) $\left\lfloor\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rfloor$, then the edge labels are distinct. In this case f is called a Lehmer -3 Mean labeling of G. In this paper we prove that some disconnected graphs are Lehmer -3 Mean graphs.

Keywords: Graph, Path, Cycle, Comb

1. Introduction

The graph considered here are simple, finite and undirected graph $G=(V,E)$ with p vertices and q edges. For a detailed survey of graph labeling we refer to Gallain [1]. For all other standard terminology and notations we follow Harary [2]

Definition1.1
The union of two graphs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ is a graph $G=G_1\cup G_2$ with vertex set $V=V_1\cup V_2$ and the edge set $=E_1\cup E_2$.
Definition1.2
The Corona of two graphs G_1 and G_2 is the graph $G=G_1\odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

2. Main Results

Theorem: 2.1

nk_3 is a Lehmer-3 mean graph for $n\geq 1$

Proof:

let the vertex set of nk_3 be $V=V_1\cup V_2\cup \ldots\ldots\ldots.\cup V_n$ where $V_i=\{v_i^1,v_i^2,v_i^3\}$ and the edge set $E=E_1\cup E_2\cup\ldots\ldots\cup E_n$ where $E_i=\{e_i^1,e_i^2,e_i^3\}$. A function $f:(V(nK_3))\rightarrow\{1,2,\ldots\ldots..q+1\}$ is defined by

$$f(u_i^j)=3(i-1)+j \quad ;1\leq i\leq n,\ \ 1\leq j\leq 3$$

In general the Lehmer -3 mean of two consecutive integers a and a+1 lies between a and a+1

consider a graph with vertices whose labels are 3i-2,3i-1 and 3i

The label of the edge joining the vertices 3i-2 and 3i-1 is 3i-2

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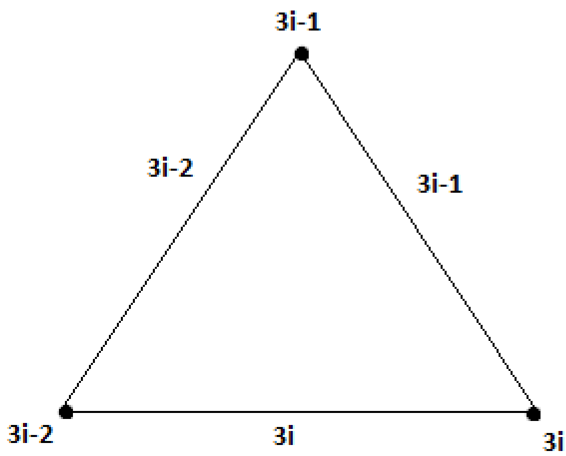
The edge joining the vertices 3i-1 and 3i is 3i-1

Similarly for the vertices 3i and 3i-2 is 3i



Thus the above labeling nK_3 is a Lehmer-3 mean graph.

Example: 2.2

Consider the graph $4K_3$

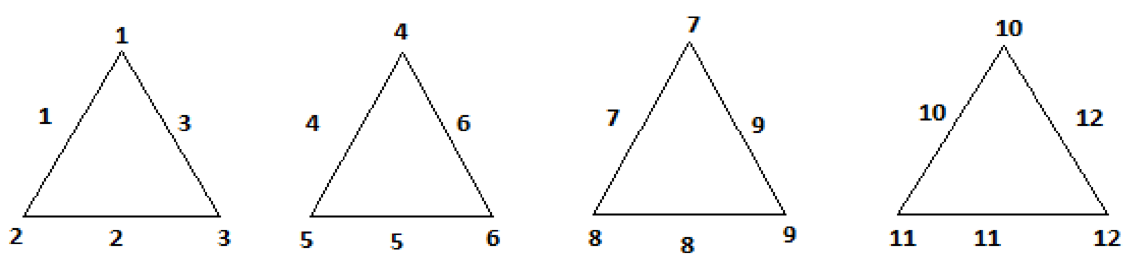


Figure -1

$4K_3$ is a Lehmer -3 mean graph.

Theorem: 2.3

$nK_3 \cup P_m$ is a Lehmer -3 mean graph for m,n≥1

Proof:

let the vertex set of nk_3, be $V=V_1 \cup V_2 \cup, V_3 \cup, \ldots\ldots \cup V_n$ where $Vi=\{v_i^1, v_i^2, v_i^3\}$ and the edge set $E=E_1 \cup E_2 \cup \ldots\ldots \cup E_n$ where $E_i=\{e_i^1, e_i^2, e_i^3\}$.

Let P_m be the path with m vertices $v_1, v_2, v_3 \ldots\ldots v_m$.

$nk_3 \cup P_m$ has 3n+m-1 edges and 3n+m vertices.

Define a function $f:V(nK_3 \cup P_m) \rightarrow \{1,2,\ldots\ldots q+1\}$ by

$$f(V_i^j)=3(i-1)+j \;;\; 1 \le i \le n$$

$$f(U_k)=3n+k \quad ;\; 1 \le k \le m$$

The label of the edges

$$f(V_iV_{i+1})=3(i-1)+j \quad ; \quad 1 \le i \le n \;, 1 \le j \le m$$

$$f(u_ku_{k+1})=3n+k \quad ; \quad 1 \le k \le n$$

Then the set of labels of the edge of nK_3 is $\{1,2,3,\ldots..3n\}$

The set of labels of the edge of p_m is $\{3n+1, 3n+2, \ldots\ldots 3n+m-1\}$

Hence $nk_3 \cup P_m$ is a Lehmer-3 mean graph m,n≥1

Example: 2.4

Consider the graph $4K_3 \cup P_5$ which has 17 vertices and 16 edges forms a Lehmer-3 mean graph.

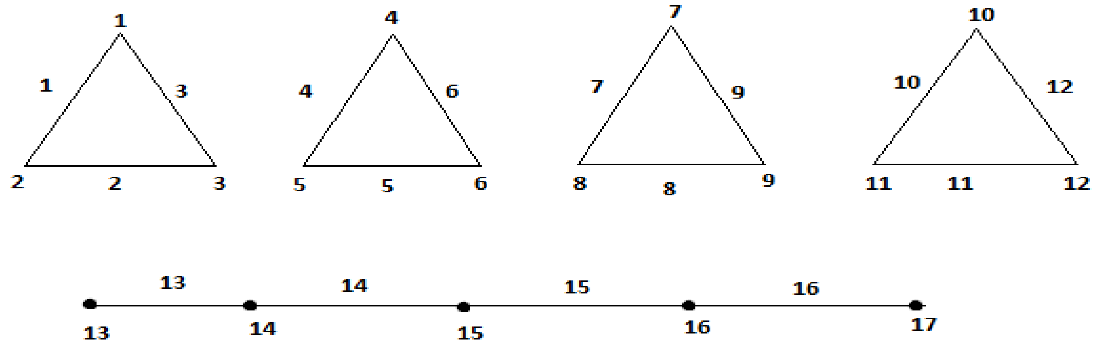


Figure -2

Theorem: 2.5

$nK_3 \cup C_m$ is a Lehmer-3 mean graph for $n \geq 1$, $m \geq 3$.

Proof:

Let the vertex set of nK_3 be $V=V_1 \cup V_2 \cup \ldots\ldots \cup V_n$ where $V_i=\{v_i^1,v_i^2,v_i^3\}$ and the edge set $E=E_1 \cup E_2 \cup \ldots\ldots\ldots \cup E_n$ where $Ei=\{e_i^1,e_i^2,e_i^3]$

Let C_m be the cycle $u_1,u_2,\ldots\ldots\ldots u_m u_1$,$nk_3 \cup C_m$ be the graph with 3n+m vertices and the same number of edges.

Define a function $f:V(nk_3 \cup C_m) \rightarrow \{1,2,\ldots\ldots 3n+m\}$ as

$$f(u_i^j)=3(i-1)+j \quad ; \quad 1 \leq i \leq n, 1 \leq j \leq 3$$

$$f(u_k)=3n+k \quad ; \quad 1 \leq k \leq m$$

Then we get a distinct edge labels for nK_3 as $\{1,2,\ldots\ldots 3n\}$ and the distinct edge labels for C_m as $\{3n+1,3n+2,\ldots\ldots\ldots 3n+m\}$

Thus $nk_3 \cup C_m$ is a Lehmer-3 mean graph.

Example: 2.6

Consider $3K_3 \cup C_6$ which has 15 vertices and 15 edges which forms a Lehmer-3 mean graph.

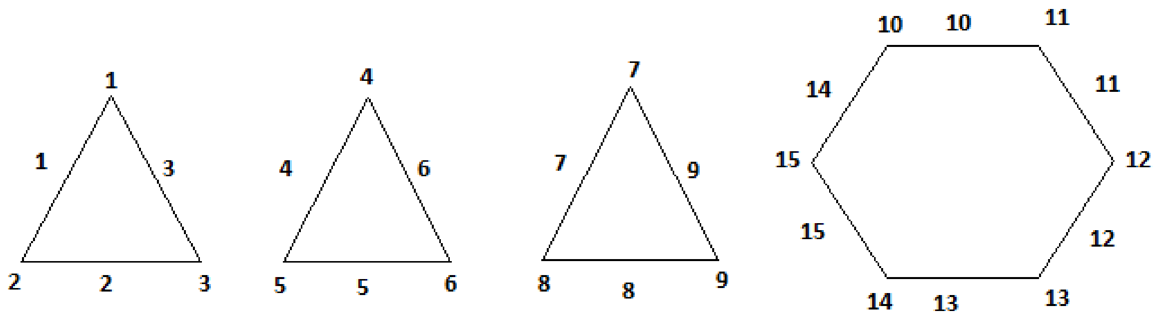


Figure - 3

Theorem: 2.7

$mK_3 \cup (P_n \odot k_1)$ be a Lehmer-3 mean graph

Proof:

Let G be a graph obtained from the union of m K_3 and $(P_n \odot K_1)$
Let K_3 be a complete graph with 3 vertices
Let $(P_n \odot K_1)$ be a comb with vertices as $v_1, v_2 \ldots v_n$; $w_1, w_2 \ldots w_n$ respectively
Define a function f: V(G) → {1,2,… q+1} defined by

$$f(u^j_i) = 3(i-1) + j \quad ; \quad 1 \le i \le m,\ 1 \le j \le 3$$
$$f(v_k) = 3m + (2k-1) \quad ; \quad 1 \le k \le n$$
$$f(w_k) = 3m + 2k \quad ; \quad 1 \le k \le n$$

Thus we obtain distinct edge labels.

Hence m $K_3 \cup (P_n \odot K_1)$ be a Lehmer-3 mean graph

Example: 2.8

$5K_3 \cup (P_5 \odot K_1)$ is a Lehmer-3 mean graph

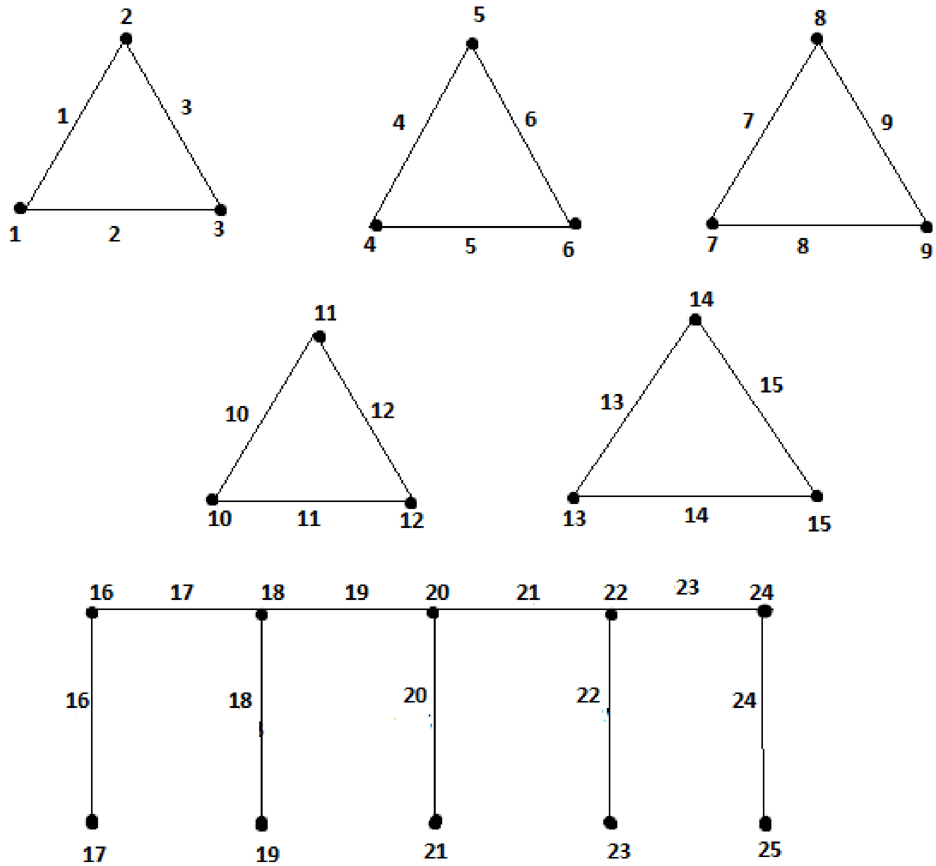


Figure-4

Theorem: 2.9

$nK_3 \cup (P_m \odot k_{1,2})$ be a Lehmer-3 mean graph

Proof:

Let G be a graph obtained from the union of nK_3 and $(P_m \odot K_{1,2})$
Let K_3 be a comlete graph with 3 vertices
Let $(P_m \odot K_{1,2})$ be a graph with vertices $v_1, v_2 \ldots v_m$; $w_1, w_2 \ldots w_m$; $x_1, x_2 \ldots x_m$ respectively
Define a function f: V(G) $\rightarrow$ {1,2,… q+1} defined by

$$f(u^j_i) = 3(i-1) + j \quad ; \quad 1 \le i \le n,\ 1 \le j \le 3$$
$$f(v_k) = 3n + (3k-2) \quad ; \quad 1 \le k \le m$$
$$f(w_k) = 3n + (3k-1) \quad ; \quad 1 \le k \le m$$
$$f(x_k) = 3n + 3k \quad ; \quad 1 \le k \le m$$

Thus we obtain distinct edge labels.

Hence $nK_3 \cup P_m \odot K_{1,2})$ is a Lehmer-3 mean graph

Example: 2.10

$5K_3 \cup (P_4 \odot K_{1,2})$ is a Lehmer-3 mean graph

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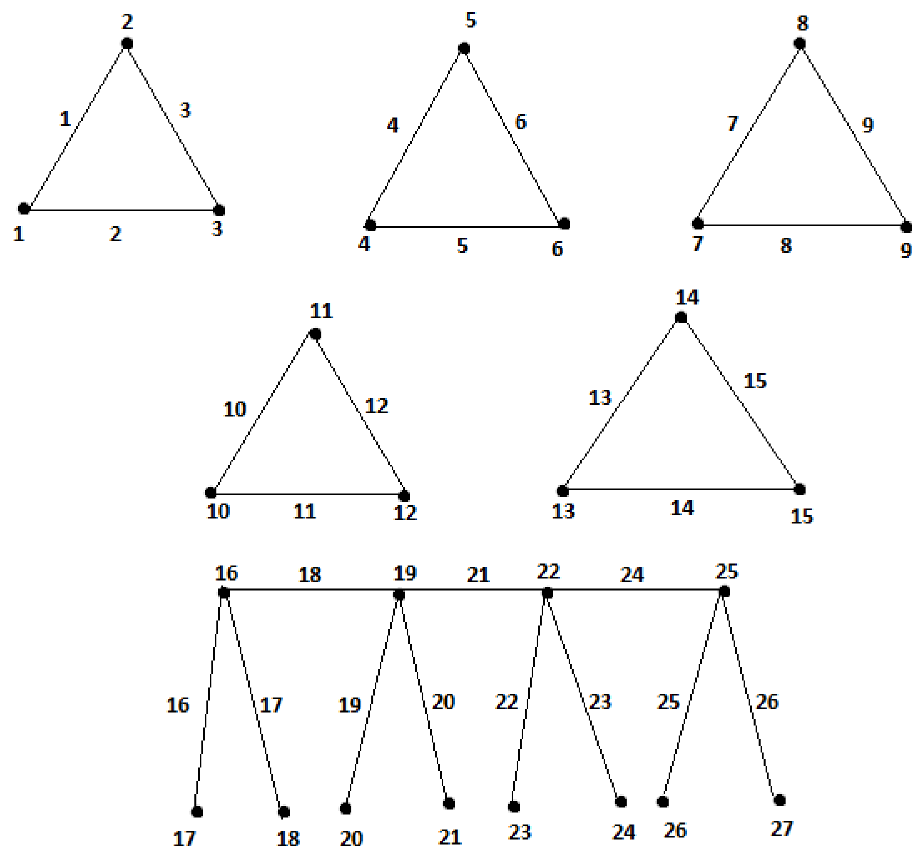

Figure-5

Theorem: 2.11

$nK_3 \cup (P_m \odot k_{1,3})$ be a Lehmer-3 mean graph

Proof:

Let G be a graph obtained from the union of nK_3 and $(P_m \odot K_{1,3})$
Let K_3 be a complete graph with 3 vertices
Let $(P_m \odot K_{1,3})$ be a graph with vertices $v_1,v_2\ldots v_m$; $w_1,w_2\ldots w_m$; $x_1,x_2\ldots x_m$; $y_1,y_2\ldots y_m$ respectively
Define a function $f: V(G) \rightarrow \{1,2,\ldots q+1\}$ defined by

$$f(u^j_i) = 3(i-1) + j \quad ; \quad 1 \le i \le n,\ 1 \le j \le 3$$
$$f(v_k) = 3n + (4k-3) \quad ; \quad 1 \le k \le m$$
$$f(w_k) = 3n + (4k-2) \quad ; \quad 1 \le k \le m$$
$$f(x_k) = 3n + (4k-1) \quad ; \quad 1 \le k \le m$$
$$f(y_k) = 3n + 4k \quad ; \quad 1 \le k \le m$$

Hence the distinct edge labeling are obtain.

Hence $nK_3 \cup (P_m \odot K_{1,3})$ is a Lehmer-3 mean graph

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Example: 2.12

$5K_3 \cup (P_4 \Theta K_{1,3})$ is a Lehmer-3 mean graph

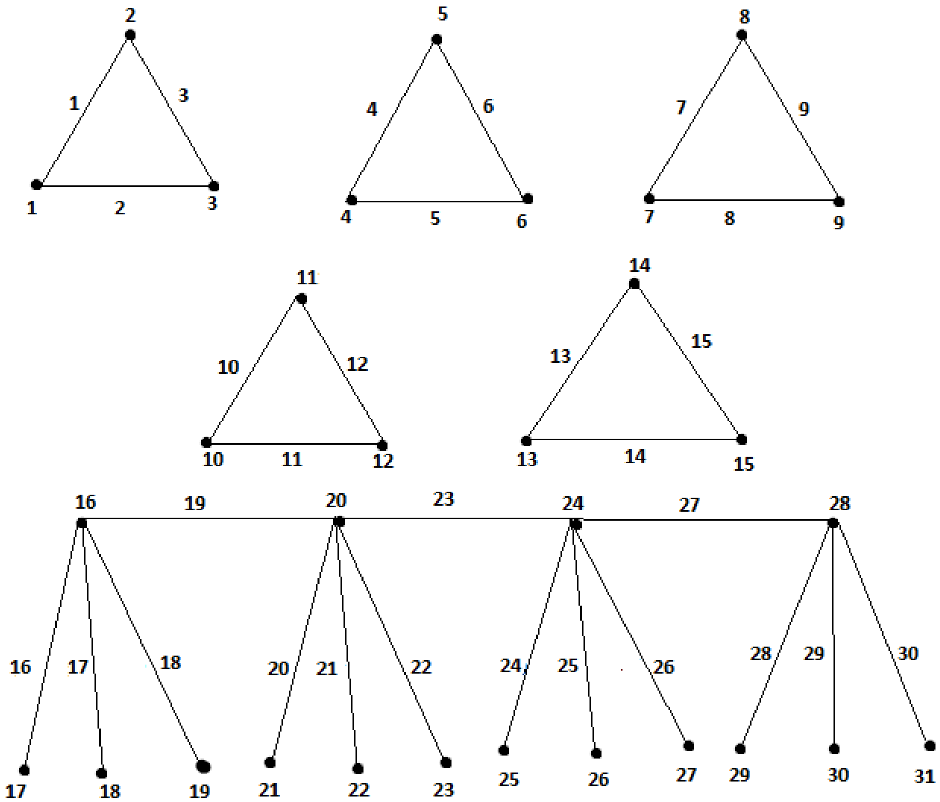

Figure- 6

Theorem: 2.13

$nK_3 \cup (P_m \Theta k_3)$ be a Lehmer-3 mean graph

Proof:

Let G be a graph obtained from the union of n times K_3 and $(P_m \Theta K_3)$
Let K_3 be a complete graph with 3 vertices
Let $(P_m \Theta K_3)$ be a graph with vertices $v_1, v_2 \ldots v_m$; $w_1, w_2 \ldots w_m$; $x_1, x_2 \ldots x_m$ respectively
Define a function $f: V(G) \rightarrow \{1,2,\ldots q+1\}$ defined by

$$
\begin{aligned}
f(u^j_i) &= 3(i-1) + j \quad ; \quad 1 \le i \le n,\ 1 \le j \le 3 \\
f(v_k) &= 3n + (4k-3) \quad ; \quad 1 \le k \le m \\
f(w_k) &= 3n + (4k-2) \quad ; \quad 1 \le k \le m \\
f(x_k) &= 3n + (4k-1) \quad ; \quad 1 \le k \le m
\end{aligned}
$$

Hence we obtain distinct edge labeling.

Hence $nK_3 \cup (P_m \Theta K_3)$ is a Lehmer-3 mean graph

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Example: 2.14

$5K_3 \cup (P_5 \odot K_3)$ is a Lehmer-3 mean graph

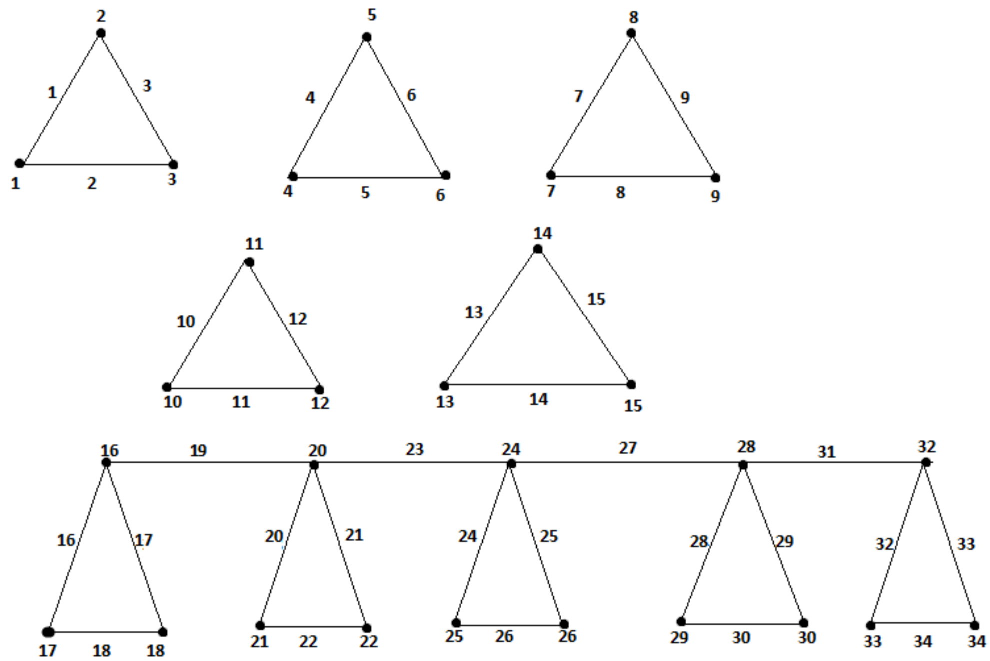

Figure-7

Theorem:2.15

$nK_3 \cup (C_m \odot k_1)$ be a Lehmer-3 mean graph

Proof:

Let G be a graph obtained from the union of nK_3 and $(C_m \odot K_1)$
Let K_3 be a complete graph with 3 vertices respectively
Let $(C_m \odot K_1)$ be a graph with vertices $v_1, v_2 \ldots v_m$; $w_1, w_2 \ldots w_m$ respectively
Define a function $f: V(G) \rightarrow \{1,2,\ldots q+1\}$ defined by

$$f(u^j_i) = 3(i-1) + j \quad ; \quad 1 \leq i \leq n,\ 1 \leq j \leq 3$$
$$f(v_k) = 3n + (2k-1) \quad ; \quad 1 \leq k \leq m$$
$$f(w_k) = 3n + 2k \quad ; \quad 1 \leq k \leq m$$

Thus we obtain distinct edge labels.

Hence $nK_3 \cup (C_m \odot K_1)$ is a Lehmer-3 mean graph

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Example: 2.16
$5K_3 \cup (P_6 \odot K_1)$ is a Lehmer-3 mean graph

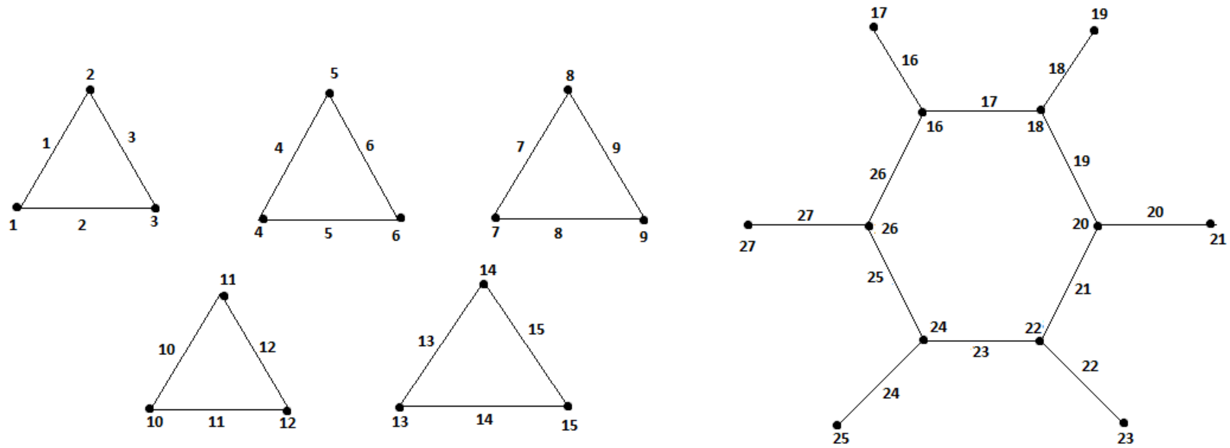

Figure- 8

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