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FIXED ORDINARY POINTS ARE POINTS COMPATIBLE WITH DIFFERENT TYPES

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ABSTRACT:

In this article, a single common point has been demonstrated between two sets of compatible P-type maps over a complete metric area and a common theory of common points for four configurations over a metric area. These theories are a motivating movement around the theories that circulate Banach Fixed. We demonstrate some common fixed point theories for a couple of weak compatible designations in vague metric spaces in both Kramosil and Michalek, which means that George and Veeramani use the new feature and give some examples. Our results improve and generalize Mihet's key findings in (Mihet, 2010) and several fixed point theories in vaguely metric spaces. We introduce concepts of compatible designations of type (R), type (K) and type (E) in double metric spaces and demonstrate some common fixed point theories for these assignments

Keywords: Fixed point, self maps, compatible mappings, compatible mappings of type (P), associated sequence.

1. INTRODUCTION:

The notion of fuzzy sets was introduced by Zadeh [1] in 1965. Since that time a substantial literature has developed on this subject; see, for example, [2–4]. Fixed point theory is one of the most famous mathematical theories with application in several branches of science, especially in chaos theory, game theory, nonlinear programming, economics, theory of differential equations, and so forth. The works noted in [5–7] are some examples from this line of research.

Fixed point theory in fuzzy metric spaces has been developed starting with the work of Heilpern .He introduced the concept of fuzzy mappings and proved some fixed point theorems for fuzzy contraction mappings in metric linear space, which is a fuzzy extension of the Banach's contraction principle. Subsequently several authors have studied existence of fixed points of fuzzy mappings. Butnariu also proved some useful fixed point results for fuzzy mappings. Badshah and Joshi studied and proved a common fixed point theorem for six mappings on fuzzy metric spaces by using notion of semicompatibility and reciprocal continuity of mappings satisfying an implicit relation.

For the reader's convenience we recall some terminologies from the theory of fuzzy metric spaces, which will be used in what follows.

Let X be a non empty set, a mapping $\rightarrow$S X X : is called a self map of X, if there is an element $\in$x X such that =S x x ( ) , then x is called a fixed point of the self map S of X . Suppose S and T are self maps of X, if $\in$x X is such that = =Sx Tx x , then x is called a common fixed point of S and T. Two self maps S and T are said to be commutative if . A result giving a set of conditions on S and X under which S has a fixed point is=ST TS known as fixed point theorem. In recent times fixed point theorems have gained importance because of their numerous applications. It is well known that the classical Banach Contraction Principle is the first ever fixed point theorem. Later on this result was generalized and extended in various ways by many authors: for instant Das and Naik (1979), Fisher (1981), Khan and Imbad (1983), Kang and Kim (1992) etc., proved some common fixed point theorems . Fixed point theorem has many applications in various fields like Differential Equations, Operator Theory, Game Theory and Economics etc. Gerald Jungck initiated the concept of compatibility. In the later years the concept of compatibility was further generalized in many ways. introduced the concept of compatible mappings of type (A) and they gave some examples to show that compatible mappings of type (A) need not be compatible mappings. Extending type (A)] introduced the concept of compatible mappings of type (B) and they gave some examples to show that compatible mappings of type (B) need not be compatible mappings of type (A) introduced the concept of compatible mappings of type (P)] introduced another extension of compatible mappings of type (A) in normed spaces called compatible mappings of type (C) and with some examples they compared these mappings with compatible mappings. From the propositions given in and we observe that the concept of compatible, compatible maps of type (A), compatible maps of type (B) compatible maps of type (P) and compatible maps of type (C) are equivalent when S and T are continuous. Singh introduced the concept of compatible maps of the type (E) on a metric space, which is equivalent to the concept of compatible maps (compatible maps of type (A), compatible maps of type (P)) under some conditions. It has been known from the paper of Khannan [6] that there exists maps that have discontinuity in the domain but have fixed points. Moreover, the maps involved in every case were continuous at the fixed point introduced the notion of weakly compatible and showed that compatible maps are weakly compatible but not conversely introduced the concept of compatible− α mappings and compatible maps of type (A) and generalized the results of in a new setting. Recently, in 2011 M. R. Singh & introduced the concept of and−S compatible maps of the type (E)−A Several researches proved the fixed point theorems in metric spaces, Banach spaces, Topological spaces based on the Banach contraction principle has proved fixed point theorems for two self maps using compatible condition has proved fixed point theorems of a complete metric space by using compatibility condition has proved certain common fixed point theorem by using compatibility of type (P) and reciprocal continuity has proved fixed point theorems by using weakly compatible condition. Swathi Mathur has proved fixed point theorems in metric space by using weaker compatibility conditions, compatible mappings of type (C) and compatible mappings of type (C).− α

2. INTRODUCTION AND PRELIMINARIES

Let X be a nonempty set. Multiplicative metric is a mapping d : X × X → R + satisfying the following conditions: (i) d(x, y) ≥ 1 for all x, y ∈ X and d(x, y) = 1 if and only if x = y; (ii) d(x, y) = d(y, x) for all x, y ∈ X; (iii) d(x, y) ≤ d(x, z) · d(z, y) for all x, y, z ∈ X (multiplicative triangle inequality).

Let (X, d) be a multiplicative metric space. Then a sequence {xn} in X said to be (1) a multiplicative convergent to x if for every multiplicative open ball B(x) = {y | d(x, y) < }, > 1, there exists a natural number N such that n ≥ N, then xn ∈ B(x), that is, d(xn, x) → 1 as n → ∞. (2) a multiplicative Cauchy sequence if for all > 1, there exists N ∈ N such that d(xn, xm) < for all m, n > N, that is, d(xn, xm) → 1 as n → ∞. (3) We call a multiplicative metric space complete if every multiplicative Cauchy sequence in it is multiplicative convergent to x ∈ X.

3. DEFINITIONS AND PRELIMINARIES

1. COMPATIBLE MAPPINGS

Two self maps S and T of a metric space (X,d) are said to be compatible mappings if n ∞→ limd(STxn,TSxn)=0, whenever is a sequence in X such that n ∞→ limSxn= n ∞→ limTxn= t for some t∈X.

A. Compatible Mappings of Type Two self maps S and T of a metric space (X, d) are said to be compatible mappings of type(A) if n ∞→ lim d(STxn ,TTxn) = 0 and n ∞→ lim d(TSxn,SSxn) = 0 whenever is a sequence in X such that n ∞→ lim Sxn = n ∞→ lim Txn = t for some t∈X.

B. Compatible Mappings of Type Two self maps S and T of a Metric Space(X,d) are said to be compatible mappings of type(B) if n ∞→ limd(STxn ,TTxn) ≤ 2 1 [n ∞→ limd(STxn ,St) + n ∞→ limd(St, SSxn)] and n ∞→ limd(TSxn, SSxn) ≤ 2 1 [n ∞→ limd(TSxn,Tt) + n ∞→ lim d(Tt, TTxn)] whenever is a sequence in X such that n ∞→ limSxn = n ∞→ limTxn = t for some t∈X .

C. COMPATIBLE MAPPINGS OF TYPE (P)

Two self maps S and T of a Metric Space (X,d) are said to be compatible mappings of type (P) if n ∞→

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lim d(SSxn,TTxn) = 0, when ever is a sequence in X such that n ∞→ lim Sxn = n ∞→ limTxn = t for some t∈X. It is clear that every compatible pair is weakly compatible but its converse need not be true.

4. PROPERTIES OF COMPATIBLE MAPPINGS OF TYPES

Introduced the notion of compatible mappings and its variants in a multiplicative metric space. Now we introduce the notions of compatible mappings of types in the setting of a multiplicative metric space as follows:

Let f and g be two mappings of a multiplicative metric space (X, d) into itself. Then f and g are called (1) compatible of type (R) if limn→∞ d(fgxn, gfxn) = 1 and limn→∞ d(ffxn, ggxn) = 1, whenever {xn} is a sequence in X such that limn→∞ fxn = limn→∞ gxn = t for some t ∈ X. (2) compatible of type (K) if limn→∞ d(ffxn, gt) = 1 and limn→∞ d(ggxn, f t) = 1, whenever {xn} is a sequence in X such that limn→∞ fxn = limn→∞ gxn = t for some t ∈ X. (3) compatible of type (E) if limn→∞ ffxn = limn→∞ fgxn = gt and limn→∞ ggxn = limn→∞ gfxn = f t, whenever {xn} is a sequence in X such that limn→∞ fxn = limn→∞ gxn = t for some t ∈ X.

Let f and g be two mappings of a multiplicative metric space (X, d) into itself. Then f and g are called reciprocally continuous if limn→∞ fgxn = f t and limn→∞ gfxn = gt, whenever {xn} is a sequence in X such that limn→∞ fxn = limn→∞ gxn = t for some t ∈ X. Now we give some properties related to compatible mappings of type (R) and type (E).

. Let f and g be compatible mappings of type (R) of a multiplicative metric space (X, d) into itself. If f t = gt for some t ∈ X, then fgt = ff t = ggt = gf t. Proof. Suppose that {xn} is a sequence in X defined by xn = t, n = 1, 2, . . . for some t ∈ X and f t = gt. Then we have fxn, gxn → f t as n → ∞. Since f and g are compatible of type (R), we have d(fgt, gf t) = limn→∞ d(fgxn, gfxn) = 1. Hence we have fgt = gf t. Therefore, since f t = gt, we have fgt = ff t = ggt = gf t. This completes the proof.

Let f and g be compatible mappings of type (R) of a multiplicative metric space (X, d) into itself. Suppose that limn→∞ fxn = limn→∞ gxn = t for some t ∈ X. Then (a) limn→∞ gfxn = f t if f is continuous at t. (b) limn→∞ fgxn = gt if g is continuous at t. (c) fgt = gf t and f t = gt if f and g are continuous at t. Proof. (a) Suppose that f is continuous at t. Since limn→∞ fxn = limn→∞ gxn = t for some t ∈ X, we have ffxn, fgxn → f t as n → ∞. Since f and g are compatible of type (R), we have limn→∞ d(gfxn, f t) = limn→∞ d(gfxn, fgxn) = 1. Therefore, limn→∞ gfxn = f t. (a) holds. (b) The proof of limn→∞ fgxn = gt follows by similar arguments as in (a). (c) Suppose that f and g are continuous at t and {xn} is a sequence in X defined xn = t (n = 1, 2, . . .) for some t ∈ X Since gxn → t as n → ∞ and f is continuous at t, by (a), gfxn → f t as n → ∞. On the other hand, g is also continuous at t, gfxn → gt. Thus, we have f t = gt by the uniqueness of limit and so by Proposition 2.3, fgt = gf t. This completes the proof.

Theorem 1. Let A, B, S and T be mappings from a complete metric space (X, d) into itself satisfying the conditions Suppose that one of A, B, S and T is continuous the pairs A, S and B, T are compatible on X. Then A, B, S and T have a unique common fixed point in X

Proof. Let {yn} be the sequence in X defined by {yn} is a Cauchy sequence and hence it converges to some point z ∈ X. Consequently, the subsequences {Ax2n}, {Sx2n}, {Bx2n−1} and {T x2n−1} of {yn} also converge to the point z. Now, suppose that S is continuous. Since A and S are compatible on X, Lemma gives that S 2x2n −→ Sz, ASx2n −→ Sz as n → ∞.

d(ASx2n, Bx2n−1) ≤ p max

{ d(ASx2n, S2x2n), d(Bx2n−1, T x2n−1),

1/ 2 [d(ASx2n, T x2n−1) + d(Bx2n−1, S2x2n)], d(S 2x2n, T x2n−1)}

+ q max {d(ASx2n, S2x2n), d(Bx2n−1, T x2n−1)}

+ r max { d(ASx2n, T x2n−1), d(Bx2n−1, S2x2n)} .

Letting n → ∞, we have

d(Sz, z) ≤ p max { 0, 0, 1 /2 [d(Sz, z) + d(z, Sz)], d(Sz, z)}

so that z = Sz

d(Az, Bx2n−1) ≤ p max { d(Az, Sz), d(Bx2n−1, T x2n−1), 1 /2 [d(Az, T x2n−1) + d(Bx2n−1, Sz)], d(Sz, T x2n−1) + q max { d(Az, Sz), d(Bx2n−1, T x2n−1) + r max { d(Az, T x2n−1), d(Bx2n−1, Sz) }.

Letting n → ∞, we have d(Az, z) ≤ p max { d(Az, Sz), 0, 1 /2 [d(Az, z) + d(z, Sz)], d(Sz, z) + q d(Az, Sz) + r max { d(Az, z), d(z, Sz) , so that z = Az. Since A(X) ⊂ T(X), we have z ∈ T(X) and hence there exists a point u ∈ X such that z = Az = T u. d(z, Bu) = d(Az, Bu) ≤ p max { 0, d(Bu, T u), 1/ 2 [d(Az, T u) + d(Bu, z)], d(Sz, T u) + q d(Bu, T u) + r max {d(Az, T u), d(Bu, z) , which implies that z = Bu. Since B and T are compatible on X and T u = Bu = z, we have d(T Bu, BT u) = 0 by Lemma 2.6 and hence T z = T Bu = BT u = Bz. Moreover,

d(z, T z) = d(Az, Bz) ≤ p max { 0, d(Bz, T z), 1 2 [d(z, T z) + d(Bz, z)], d(z, T z) + q d(Bz, T z) + r max {d(z, T z), d(Bz, z)} ,

5. CONCLUSION:

1. To define compatible of type (A)– α

2. To prove fixed point theorems in metric space by using compatible mapping of type (A) and compatible mapping of type (A)– α

3. To define compatible mapping of type (X) and compatible mapping of type (X)– α

4. To prove fixed point theorems in metric space by using compatible mapping of type (X) and compatible mapping of type (X).– α

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