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HEAT CONDUCTED THROUGH A PARABOLIC FIN VIA MEANS OF ELZAKI TRANSFORM

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Abstract
Fins or spines are the extended surfaces used in improving the heat transfer rate through the conducting material and are mostly used in the heat exchanging devices. The paper inquires the theory of a parabolic fin to find the temperature distribution along its length and hence the rate of conduction of heat through it via means of Elzaki Transform. The purpose of paper is to prove the applicability of Elzaki Transform means to find the temperature distribution along the length of the parabolic fin and hence the rate of conduction of heat through it.

Index terms: Parabolic fin, Elzaki Transform means, temperature distribution, rate of conduction of heat.

1. Introduction

Heat transfer is a matter of outspread interest to the engineers and technicians committed in the design, construction, and operation of equipment used to transfer heat in the scientific and industrial technology [1, 2]. Fins or spines are the extended surfaces used to improve the heat transfer rate from the conducting material [3] and are mostly used in the heat exchanging devices [4] like computer central processing unit, power plants, radiators, heat sinks, etc. The paper inquires the theory of a parabolic fin to find the temperature distribution along its length and hence the rate of conduction of heat through it via Elzaki Transform means. Elzaki Transform means has been applied in solving boundary value problems in most of the science and engineering disciplines [5, 6]. It also comes out to be very effective tool to find the temperature distribution along the length of a parabolic fin and hence the rate of conduction of heat through it.

2. Basic Definitions

2.1 Elzaki Transform

If the function ɦ(y), $y \geq 0$ is having an exponential order and is a piecewise continuous function on any interval, then the Elzaki transform [5, 6] of ɦ(y) is given by

$$E\{ɦ(y)\} = \mathcal{H}(p) = p\int_0^\infty e^{-\frac{y}{p}} ɦ(y)dy.$$

2.2 Inverse Elzaki Transform

If we write E [ɦ(y)] = $\mathcal{H}(p)$, then inverse Elzaki transform [5, 6] of the function $\mathcal{H}(p)$ is given by $E^{-1}[\mathcal{H}(p)] =$ ɦ(y), where E^{-1} is called the inverse Elzaki transform operator. The Inverse Elzaki Transform of Some of the Functions are given by

- $E^{-1}\{p^n\} = \frac{y^{n-2}}{(n-2)!}$, $n = 2, 3, 4 \ldots$
- $E^{-1}\{\frac{p^2}{1-ap}\} = e^{ay}$
- $E^{-1}\{\frac{p^3}{1+a^2p^2}\} = \frac{1}{a}\sin ay$
- $E^{-1}\{\frac{p^2}{1+a^2p^2}\} = \frac{1}{a}\cos ay$

2.3 Elzaki Transform of Derivatives

The Elzaki Transform of some of the Derivatives [5, 6] of h(y) are given by

- $E\{h'(y)\} = \frac{1}{p}E\{h(y)\} - p\,h(0)$
  $or\ E\{ɦ'(y)\} = \frac{1}{p}\mathcal{H}(p) - p\,ɦ(0),$
- $E\{ɦ''(y)\} = \frac{1}{p^2}\mathcal{H}(p) - ɦ(0) - pɦ'(0),$
- $E\{yɦ'(y)\} = p^2\frac{d}{dp}\left\{\frac{1}{p}\mathcal{H}(p) - p\,ɦ(0)\right\} - p\left\{\frac{1}{p}\mathcal{H}(p) - p\,ɦ(0)\right\},$

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- $E\{y\, \hbar''(y)\} = p^2 \frac{d}{dp}\left\{\frac{1}{p^2}\mathcal{H}(p) - \hbar(0) - p\hbar'(0)\right\} - p\left\{\frac{1}{p^2}\mathcal{H}(p) - \hbar(0) - p\hbar'(0)\right\}$.

3. Material and Method

The differential equation for analyzing a parabolic fin [4, 7] (assuming that heat flow is one dimensional) is given by

$$x^2\theta''(x) + 2x\theta'(x) - M^2 l^2 \theta(x = 0 \ldots .(1),$$

$$\theta' \equiv \frac{d}{dx}$$

$where\ M = \sqrt{\frac{2h}{tk}}$, l is the length of the fin between the base at x = l and the tip at x = 0, t is the thickness of the fin which increases uniformly from zero at the tip to t at the base, k is thermal conductivity, h is the coefficient of transfer of heat by convection, $\theta(x) = T(x) - T_s$ is the temperature difference, T_s is the temperature of the environment of the fin and T_0 is the temperature at the base x = 0 of the fin.

Substituting $x = e^z$, the equation (1) can be rewritten into a form:

$$\theta''(x) + \theta'(x) - M^2 l^2 \theta(x) = 0 \ldots (2)$$

$$\theta' \equiv \frac{d}{dz}$$

The Elzaki transform of (2) gives

$$\left[\frac{1}{p^2}\theta(p) - \theta(0) - p\theta'(0)\right] + \frac{1}{p}\,\theta(p) - p\theta(0) - M^2 l^2 \theta(p) = 0 \ldots .. (3)$$

Put $\theta(0) = C\ and\ \theta'(0) = D$, and simplifying and rearranging (3), we get

$$\theta(p) = \frac{(1 + C)p + pD}{\frac{1}{p^2} + \frac{1}{p} - M^2 l^2}$$

Or

$$\theta(p) = \frac{(1 + C)p + pD}{\left(\frac{1}{p} - c_1\right)\left(\frac{1}{p} + c_2\right)} \ldots (4)$$

$where\ c_1 = \frac{-1+(1+4M^2l^2)^{1/2}}{2}\ and\ c_1 = \frac{-1-(1+4M^2l^2)^{1/2}}{2}$.

This equation (4) can be rewritten as

$$\theta(p) = [(1 + c_1)C + D]\frac{p^2}{(1 - c_1 p)}$$

$$+[(c_2 - 1)C - D]\frac{p^2}{(1 + c_2 p)} \ldots .(5)$$

The inverse Elzaki transform of (5) provides

$$\Theta(x) = [(1 + c_1)C + D]\, e^{c_1 z}$$

$$+[(c_2 - 1)C - D]e^{-c_2 z}$$

Or

$$\Theta(x) = [(1 + c_1)C + D]\, x^{c_1} + [(c_2 - 1)C - D]x^{-c_2} \ldots . (6)$$

As $\Theta(0)\ is\ finite$ [4, 7], $therefore,\ the\ term$ $[(c_2 - 1)C - D]x^{-c_2}\ is\ equated\ to\ zero.$

$i.e. [(c_2 - 1)C - D]x^{-c_2} = 0, which\ gives$

$$D = (c_2 - 1)\mathrm{C}$$

From (6), we have

$$\Theta(x) = C(c_1 + c_2)\, x^{c_1} \ldots . (7)$$

To find the constant C, at x = l, $\Theta(l) = \Theta_0$ [4, 7], therefore, from (7)

$$\mathrm{C} = \frac{1}{c_1+c_2}\, \Theta_0 l^{-c_1}$$

Hence (7) can be rewritten as

$$\Theta(x) = \Theta_0 l^{-c_1} x^{c_1}$$

Or

$$\Theta(x) = \Theta_0 (x/l)^{c_1} \ldots (8)$$

The equation (8) gives the temperature distribution along the length of the parabolic fin.

The heat conducted through the parabolic fin is given by the Fourier's Law of heat conduction [8, 9, 10,] as

$$H = \mathrm{kA}\left(\Theta'(x)\right)_{x=l} = \mathrm{kbt}\left(\Theta'(x)\right)_{x=l}$$

Using equation (8), we get

$H = \text{kbt}\,\Theta_0 c_1/l$

Or

$H = \text{kbt}\,\Theta_0 \frac{-1+(1+4M^2l^2)^{1/2}}{2l}$ ….. (9)

This equation (9) gives the expression for the rate of conduction of heat through the parabolic fin.

4. Result and Conclusion

We have found find the temperature distribution along the length of the parabolic fin and hence the rate of conduction of heat through it via Elzaki Transform means. It is found that with the increase in length of the triangular fin or parabolic fin, temperature increases and hence the rate of conduction of heat at any cross-section of the parabolic fin increases. The method has come out to be very effective tool to find temperature distribution along the length of the parabolic fin and hence the rate of conduction of heat through it.

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