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EDGE DOMATIC NUMBER AND EDGE DOMINATION NUMBER OF FUZZY DIGRAPH

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ABSTRACT:

In this paper we discuss about Edge Domatic Number and Edge Domiation Number of Fuzzy digraph. The condition for the edge dominating set being a minimal edge domination set is also discussed.The Edge Domatic Number for the complete Fuzzsy Digraph is derived.

KEYWORDS

Fuzzy digraph, Spanning fuzzy sub digraph, concept of edge domination , edge domination number, connected edge domination of fuzzy digraph ,Domatic number of Fuzzy digraph,Edge Domatic Number of Fuzzy Digraph.

I INTRODUCTION

The concept of fuzzy graph was introduced by Rosenfeld [3] in 1975. J.Cockayne, S.T.Hedetniemi,[2] (1977) introduced the concepts of the domatic number of a graph. The concept of Edge- Domatic Number of Graph is introduced by B.Zelinka in 1983[1]. The concept of domination in fuzzy graphs are introduced by A. Somasundaram and S. Somasundaram [3] in 1998. The concept of edge domination is introduced by V.R. Kulli and D.K. Patwari [6]. Proceeding in this same chain we analyze about the edge domination, connected edge domination of fuzzy digraphs and Domatic Number and Edge Domatic Number of Fuzzy Digraph.

II PRELIMINARIES

Definition 2.1:

Fuzzy digraph $\vec{\xi} = (V,\sigma,\vec{\mu})$ is a non-empty set V together with a pair of functions $\sigma : V \to [0,1]$ and $\vec{\mu} : V \times V \to [0,1]$ such that for all x, y ∈ V , $\vec{\mu}(x,y) \le \sigma(x) \wedge \sigma(y)$.Since $\vec{\mu}$ is well defined, a fuzzy digraph has at most two directed edges (which must have opposite directions) between any two vertices. Here $\vec{\mu}(u,v)$ is denoted by the membership value of the edge $\overrightarrow{(u,v)}$.The loop at a vertex x is represented by $\vec{\mu}(x,x) \neq 0$. Here $\vec{\mu}$ need not be symmetric as $\vec{\mu}(x,y)$ and $\vec{\mu}(y,x)$ may have different values. The underlying crisp graph of directed fuzzy graph is the graph similarly obtained except the directed arcs are replaced by undirected edges.

Definition 2.2:

The fuzzy sub digraph $\vec{\zeta}(V_1,\sigma',\vec{\mu}')$ is said to be spanning fuzzy sub digraph of $\vec{\xi}(V,\sigma,\vec{\mu})$ if $\sigma(u) = \sigma'(u)$ for all $u \in V_1$ and $\vec{\mu}'(u,v) \le \vec{\mu}(u,v)$ for all u, v ∈ V.

Definition2.3:

The Strength of the connectedness between two vertices (u,v) in a fuzzy Digraph $\vec{\xi}$ is $\vec{\mu}^{\infty}(u,v)=\text{Sup}\{\vec{\mu}^{k}(u,v);k=1,2,3\ldots.\}$ where $\vec{\mu}^{k}(u,v)=\text{Sup}\{\vec{\mu}(u,u_1)\wedge\vec{\mu}(u_2,u_3)\wedge\ldots..\ \vec{\mu}(u_{k-1},v)\}$. An directed arc (u,v) is said to be strong arc if $\vec{\mu}(u,v)= \vec{\mu}^{\infty}(u,v)$.If $\vec{\mu}(u,v)=0$ for every $v \in V$ then u is called isolated vertices.

Definition 2.4:

An edge $e=\overrightarrow{xy}$ of a fuzzy digraph is called an effective edge if $\vec{\mu}(x,y) = \sigma(x) \wedge \sigma(y)$

$N^{+}(x) = \{y \in V \mid \vec{\mu}(xy) = \sigma(x) \wedge \sigma(y)\}$ is called the inneighborhood of x and $N^{+}[x] = N^{+}(x) \cup \{x\}$ is the closed inneighbourhood of x.

Definition 2.5:

The effective in degree of a vertex u is defined to be sum of the weights of effective edges incident in to u and it is denoted by $Ed^{+}(u)$

Definition 2.6:

The effective out degree of a vertex u is defined to be sum of the weights of effective edges incident out of u and it is denoted by $Ed^{-}(u)$

Definition 2.7:

The minimum effective in degree $\delta E^{+}(\vec{\xi})$=min{ $Ed^{+}(u)$ / $u \epsilon V(\vec{\xi})$} and minimum effective out degree is $\delta E^{-}(\vec{\xi})$=min{ $Ed^{-}(u)$ / $u \epsilon V(\vec{\xi})$}.Then the effective degree $\delta E(\vec{\xi}) = \delta E^{+}(\vec{\xi}) + \delta E^{-}(\vec{\xi})$.

Definition 2.8:

$\vec{\xi}$ = (V,σ,$\vec{\mu}$) be a Fuzzy Digraph.A Partition on $\vec{\mu}(\vec{\xi})$ is called the Domatic Partition if all its classes are fuzzy Dominating sets of $\vec{\xi}$ .The Maximum Cardinality of a Domatic Partition of $\vec{\xi}$ is called the Domatic Number of Fuzzy Digraphs.

III EDGE DOMINATION NUMBER IN FUZZY DIGRAPHS

Definition 3.1:

Let $\vec{\xi}$ = (σ,$\vec{\mu}$) be a fuzzy digraph on (V, $\vec{X}$).A subset $\vec{S}$ of $\vec{X}$ is said to be an edge domination set in $\vec{\xi}$ if for every edge in $\vec{X}$-$\vec{S}$ is adjacent to one effective edge in $\vec{S}$.The minimum fuzzy cardinality of an edge dominating set $\vec{\xi}$ is called the edge dominating number of $\vec{\xi}$ and is denoted by $\dot{\gamma}(\vec{\xi})$.

Definition 3.2:

Let $\vec{\xi}$ = (σ,$\vec{\mu}$) be a fuzzy digraph on (V, $\vec{X}$). an edge domination $\vec{S}$ of a fuzzy digraph $\vec{\xi}$ is connected edge dominating with $<\vec{S}>$ is connected. The connected edge domination number $\dot{\gamma}_c(\vec{\xi})$ is the minimum cardinality of connected edge dominating set..

Example 3.3:

Edge dominating set of $\vec{\xi}$ ={$\overrightarrow{be}$,$\overrightarrow{ec}$} and $\dot{\gamma}(\vec{\xi})$= 0.5

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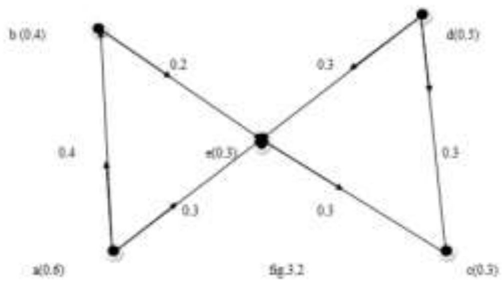

fig 3.2

Theorem 3.4:

An directed edge dominating set $\vec{S}$ is minimal if and only if for each edge $\overrightarrow{xy} \epsilon \vec{X}$

One of the following conditions holds.

$N^{+}(\overrightarrow{xy}) \cap \vec{S} = \phi$ and $N^{-}(\overrightarrow{xy}) \cap \vec{S} = \phi$

There exists an edge $\vec{e} \; \epsilon \; \vec{X}$- $\vec{S}$ such that N($\vec{e}$) $\cap \; \vec{S}$ ={ $\overrightarrow{xy}$} and $\overrightarrow{xy}$ is an effective edge.

Proof:

Let $\vec{S}$ be the minimal edge dominating set and $\overrightarrow{xy} \; \epsilon \; \vec{S}$ .then $\overrightarrow{S1} = \vec{S} - \overrightarrow{xy}$ is not an edge dominating set and hence there exist $\vec{e} \, \epsilon \vec{X} - \overrightarrow{S1}$ such that $\vec{e}$ is dominated by any element of $\overrightarrow{S1}$.

If $\vec{e} = \overrightarrow{xy}$ we get (a) and if $\vec{e} \neq \overrightarrow{xy}$ we get (b) . The converse is obvious.

Definition 3.5:

An edge $\overrightarrow{xy}$ of a fuzzy digraph is said to be an isolated edge if no effective edges incident into or incident out of the vertices x and y.

Theorem 3.6:

If $\vec{\xi}$ is a fuzzy digraph without isolated edges then for every minimal edge dominating set $\vec{S}$, $\vec{X}$-$\vec{S}$ is also an edge dominating set. ϵ N($\overrightarrow{e_1}$)

Proof:

Let $\overrightarrow{e_1}$ be any edge in $\vec{S}$. Since $\vec{\xi}$ has no isolated edges there is an edge $\overrightarrow{e_2}$ ϵ N($\overrightarrow{e_1}$). It follows from the above theorem 3.4 that $\overrightarrow{e_2} \; \epsilon \; \vec{X}$-$\vec{S}$. Then every element of $\vec{S}$ is dominated by some element of $\vec{X}$-$\vec{S}$.

IV EDGE DOMATIC NUMBER OF FUZZY DIGRAPHS

Definition 4.1:

The Edge Domatic number of Fuzzy Digraph $\vec{\xi}$ is equal to the Domatic Number of the Fuzzy Line Digraph L($\vec{\xi}$) whose vertex set is the edge set of $\vec{\xi}$ in which the two vertices are adjacent if and only if they are adjacent in $\vec{\xi}$.

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Theorem 4.2:

The Edge Domatic Number $\bar{E}d_n$ of the Fuzzy Complete Digraph $\overrightarrow{K_n}$ is equal to n.

Proof:

Let Ȅd be an Edge Dominating set in $\overrightarrow{K_n}$. The every vertex of $\overrightarrow{K_n}$ is the initial vertex of an edge from Ȅd then the Fuzzy subdigraph of $\overrightarrow{K_n}$ formed by the Edges from Ȅd Is a spasnning subgraph of $\overrightarrow{K_n}$ in which all in degrees and all the out degrees are non-zero. In those graphs the number of edges is at least ‘n’ and then Ȅd $\geq$ n. If there exists a vertex u of $\overrightarrow{K_n}$ which is not initial vertex of an edge from Ȅd Ȅd , then the in degree of u is n-1 and thus $|$ Ȅd$| \geq n-1$. If there exist a vertex u of $\overrightarrow{K_n}$ Which is not the terminal vertex of an Edge from Ȅd then $|$ Ȅd$| \geq n-1$. Therefore an Edge Dominating set in $\overrightarrow{K_n}$ has at least (n-1) elements. The graph $\overrightarrow{K_n}$ has n(n-1) edges and thus $|\bar{E}d| \leq n$.A Partition of Edges set of $\overrightarrow{K_n}$, each of whose classes is the set of all edges out going from a vertex , is an Edge Domatic Partition of $\overrightarrow{K_n}$ having’ n’ classes and hence $\bar{E}d(\overrightarrow{K_n})$=n.

V CONCLUSION

The Edge Domination Number and the Edge domatic Number of Fuzzy Digraphs are defined and some of their properties are discussed. This concept may extended to the connected Domatic Number of Fuzzy Digraphs also.

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