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ON MIXED MULTI FUZZY DIGRAPH

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Abstract:

In this paper we introduce Multi Fuzzy Digraph, Mixed Fuzzy Digraph and Mixed Multi fuzzy Digraphs.

Keywords:

Orientation of Fuzzy digraph, Multi Fuzzy Digraph, Mixed Fuzzy Digraph and Mixed Multi fuzzy Digraphs, Fuzzy Arc Strong,Edge disjoint Multi Fuzzy Digraph.

I INTRODUCTION

The concept of Fuzzy set was introduced by Zadeh [4] in 1965 and the Fuzzy Graph was introduced by Rosenfeld [1] in 1975.The Fuzzy Graphs has numerous application in handling of uncertainty existing in various fields such as Computer Science, Electrical Engineering, Networking ,etc. Multi fuzzy set was introduced by Sabu Sebastian, T.V . Ramakrishnan [5] and the Multi fuzzy Graph was introduced by A.Marichamy,K.Arjunan, K.L. Mruganantha Prasad [2] .In this Paper we introduced Mixed fuzzy digraph, Multi fuzzy digraph, Mixed Multi fuzzy Digraphs.

II PRELIMINARIES

Definition 2.1:

Fuzzy digraph $\vec{\xi} = (V,\sigma,\vec{\mu})$ is a non-empty set V together with a pair of functions $\sigma : V \to [0,1]$ and $\vec{\mu} : V \times V \to [0,1]$ such that for all x, y $\in$ V , $\vec{\mu}(x,y) \leq \sigma(x)\wedge\sigma(y)$.Since $\vec{\mu}$ is well defined, a fuzzy digraph has at most two directed edges (which must have opposite directions) between any two vertices. Here $\vec{\mu}$ (u,v) is denoted by the membership value of the edge $\overrightarrow{(u,v)}$.The loop at a vertex x is represented by $\vec{\mu}(x,x) \neq 0$. Here $\vec{\mu}$ need not be symmetric as $\vec{\mu}(x,y)$ and $\vec{\mu}(y,x)$ may have different values. The underlying crisp graph of directed fuzzy graph is the graph similarly obtained except the directed arcs are replaced by undirected edges

Definition 2.2:

Let $\overrightarrow{\mathfrak{M}} = (v, \mu \cup \vec{\mu})$ be a Mixed Multi Fuzzy Digraph with special vertex s.

A mixed out-branching $\overrightarrow{f_s^{\,+}}$ with root s is a Spanning Fuzzy tree in the underlying undirected Multi Fuzzy Graph $\vec{\xi}$ of $\overrightarrow{\mathfrak{M}}$ with the property that there is a path from s to every other vertex v in $\overrightarrow{f_s^{\,+}}$.

Definition 2.3:

A mixed out-branching rooted at s is a mixed graph F whose underlying graph is a Fuzzy tree such that $\overleftrightarrow{F}$ contains an out Branching rooted at s.

Definition 2.4:

Two subgraphs of a Mixed Multi Fuzzy Digraph are edge-disjoin t they don't share any arcs or edges.

Definition 2.5:

A mixed fuzzy digraph having parallel arcs ,Parallel Edges as well as arcs that are parallel to edges is called mixed multi fuzzy Digraph.

III Mixed and Multi Fuzzy Digraph

Definition 3.1:

An orientation of a Fuzzy Digraph $\vec{\xi}$ is a sub digraph of $\vec{\xi}$ obtained by deleting only one arc between x and y for every pair x≠y of vertices such that both $\overline{xy}$ and $\overline{yx}$ are in $\vec{\xi}$.

Theorem 3.1:

Let $\mu(x, y)$ be an unoriented edge in a strong Mixed Fuzzy graph $\mathcal{M}$. The edge $\mu(x, y)$ can be replaced by an arc $\vec{\mu}(x, y)$ (With the same end-vertices) such that the resulting mixed Fuzzy $\mathcal{M}'$ is strong if and only if $\mu(x, y)$ is not a bridge.

Proof:

If $\mu(x, y)$ is a bridge then clearly there is no orientation of $\mu(x, y)$ that results in a strong Mixed Fuzzy Graph. Assume that $\mu(x, y)$ is not a bridge. Let $\mathcal{M}' = \mathcal{M} - \mu(x, y)$ .If $\mathcal{M}'$ is strong ,then any orientation of $\mu(x, y)$ leads to a strong mixed Fuzzy Graph. Thus assume that $\mathcal{M}'$ is not strong. Since $\mu(x, y)$ is not a bridge , $\mathcal{M}'$connected..Let $L_1, L_2, L_3 \ldots . L_k$ be strong components of $\mathcal{M}'$. Since $\mathcal{M}$ is strong there is only one terminal strong component say L_1 and only one terminal strong component say L_k.Let $\sigma(x)\,\sigma(y)$)be the end vertex of $\mu(x, y)$ belonging to L_1(L_k). By strong connectivity of $L_1, L_2, L_3 \ldots . L_k$ $\mathcal{M}' + \sigma(x)\,\sigma(y)$ is strong.

Theorem 3.2:

A directed Multi fuzzy Digraph $\dot{\mathcal{M}}$ is weekly k-linked if $\dot{\mathcal{M}}$ is K- arc strong.

Proof:

Let $\sigma(x_1), \sigma(x_2) \ldots \ldots \sigma(x_k), \sigma(y_1), \sigma(y_2), \ldots \ldots \sigma(y_k)$ be the given K- linked in $\dot{\mathcal{M}}$ .Construct a new directed multigraph $\dot{\mathcal{M}}'$ by adding a new vertex s and arcs sx_i i=1,2,…k to $\dot{\mathcal{M}}$. Since $\dot{\mathcal{M}}$ is k-arc strong it is not difficult to check that $D^{-}_{\dot{\mathcal{M}}'}(X) \geq k$ for every subset X of V. Hence by using the Edmond's branching theorem , $\dot{\mathcal{M}}'$ has arc –disjoint out-branching $F^{+}_{s,1}, \ldots \ldots \ldots . F^{+}_{s,k}$ all rooted at s. Since s has out degree K in $\dot{\mathcal{M}}'$, Each $F^{+}_{s,i}$ use the arc sx_i. Now it is clear that $F^{+}_{s,i}$ contains (x_i, y_i) path P_i and the paths P_i,……….. P_i form the required link.

Theorem 3.3: Every bridgeless connected Multi Fuuzy graph $\dot{\mathcal{M}}$ admits an orientation of strong radius atmost (rad $(\dot{\mathcal{M}}))^2$+rad $(\dot{\mathcal{M}})$.

Proof: Let $\sigma(u) \in V$ be arbitrary and let $dist_{\dot{\mathcal{M}}}(\sigma(u), V) = r$, then there is an orientation L of $\dot{\mathcal{M}}$ such that $dist_L(\sigma(u), V) \leq r^2 + r$ and $dist_L(V, \sigma(u)) \leq r^2 + r$.Since $\dot{\mathcal{M}}$ is bridgeless, every edge $\sigma(u)\,\sigma(v)$ is contained in some undirected cycle;let k(v) denote the length of a shortest cycle through $\sigma(u)\,\sigma(v)$. It is not difficult to prove that, for every $\sigma(v) \in N\,\sigma(u)$,$k(v) \leq 2r + 1$.

We claim that there is a subgraph $\dot{\mathcal{H}}$ of $\dot{\mathcal{M}}$ and an orientation ð of $\dot{\mathcal{H}}$ with the following properties:

(a) $N_{\dot{\mathcal{M}}}(\sigma(u)) \subseteq V(\dot{\mathcal{H}})$.

(b) For each $\sigma(v) \in N(\sigma(u))$, ð has a cycle C_v of length k(v) containing either $\sigma(uv)$ or $\sigma(vu)$.

(c) ð is the union of the cycles C_v.

Then, we have

$\max\{dist_ð(u, V(ð)), dist_ð(V(ð), u)\} \leq 2r$ **……………..(3.1)**

We demonstrate the above claim by constructing $\dot{\mathcal{H}}$ and ð step by step.Let $\mu(uv)$ be an edge in $\dot{\mathcal{M}}$ and let Z_v be an undirected cycle of length k(v) through $\mu(uv)$. Orient Z_v arbitrarily as adirected cycle and let C_v denote the cycle obtained this way. Set $H := Z_v$, $ð := C_v$. This completes the first step. At step i(≥ 2), we choose an edge $\mu(uw)$ such that $w \notin V(\dot{\mathcal{H}})$ and an undirected cycle $Z = w_1w_2 \ldots w_kw_1$ in G such that $w_1 = u$, $w_2 = w$, and $k = k(w)$. If no vertex in $Z_w - u$ belongs to $\dot{\mathcal{H}}$, then append the directed cycle $C_w = w1w2 \ldots w_kw1$ to ð and the cycle Z to $\dot{\mathcal{H}}$. Go to the next step. Otherwise, there is a vertex w_i $(2 \leq i \leq k)$ such that $w_i \in V(H)$ (and hence $w_i \in V(ð)$). Suppose that w_i has the least possible subscript with this property. Since $w_i \in V(ð)$, there is some neighbour v of u such that $w_i \in C_v$. (Recall that C_v is a directed cycle.) Let $C_v = v_1v_2 \ldots v_tv_1$, where $u = v_1$, $v \in \{v_2, v_t\}$, and $w_i = v_j$ for some j. By considering the converse of ð, if necessary, we may assume, without loss of generality, that $v = v_2$. Now we consider two cases.

Case 1: $w_k = v$. In this case, define the directed cycle $C_w = uw_2w_3 \ldots w_iC_v[v_{j+1}, u]$ and observe that C_w has length k(w). (Indeed, if C_w had more than k(w) arcs, the path $C_w[w_i, u]$ would be longer than the path P_2 =

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$w_i w_{i+1} \dots w_k u$. In that case, the walk $Z_v[u, v_j]P_2[w_{i+1}, u]$ containing uv would be of length less than k(v); a contradiction.) Let $Z_w := U\dot{\mathcal{M}}(C_w)$. Add C_w to ð and Z_w to $\dot{\mathcal{H}}$. Go to the next step.

Case 2: $w_k = v$. In this case, define the directed cycle C_w as follows: $C_w = C_v[u, vj]w_{i-1}w_{i-2} \dots w_2u$ and observe that C_w has length k(w) (the proof of the last fact is similar to the one given in Case 1). Let $Z_w := U\dot{\mathcal{M}}(C_w)$. Add C_w to ð and Z_w to $\dot{\mathcal{H}}$. Go to the next step. Since V ($\dot{\mathcal{M}}$) is fifinite and we add at least one new vertex to $\dot{\mathcal{H}}$ at each step, this process will terminate with the desired subgraph $\dot{\mathcal{H}}$ and its orientation ð. Thus, the claim is proved. Consider the directed multigraph ð. In $\dot{\mathcal{M}}$, contract all the vertices of ð into a new vertex u * (the operation of contraction for undirected fuzzy multigraphs is similar to that for directed fuzzy multigraphs) and call the resulting Fuzzy multigraph $\dot{\mathcal{M}}^*$. Note that $\dot{\mathcal{M}}^*$ is bridgeless and that by the property (a) of the above claim, we obtain $dist_{\dot{\mathcal{M}}^*}(u^*, V(\dot{\mathcal{M}}^*)) \le r - 1$.

By the induction hypothesis, there is an orientation L^* of $\dot{\mathcal{M}}^*$ such that

$dist_{L^*}(u^*, V(L^*)) \le r^2 - r$ and

$dist_{L^*}(V(L^*), u^*) \le r^2 - r$. **…………….(3.2)**

Consider an orientation L of $\dot{\mathcal{M}}$ obtained by combining L^* with ð and orienting the rest of the edges in $\dot{\mathcal{M}}$ arbitrarily. By (3.1) and (3.2), we have

$dist_L(u, V(L)) \le r^2 + r$ and $distL(V(L), u) \le r^2 + r$.

IV Conclusion

The Study of Multi Fuzzy Digraph, Mixed Fuzzy Digraph and Mixed Multi fuzzy Digraphs will leads to develop their properties which can be applied graphs where ever we consider both directed and undirected edges with parallel edges and self loops.

V References

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