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SOLITARY-WAVE SOLUTION OF BURGER-FISHER EQUATION THROUGH TANH-COTH METHOD

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Absract- The tanh-coth method and for finding exact travelling wave and soliton solutions of nonlinear partial differential equations are described. We consider Burger-Fisher equation travelling wave and solitary wave solutions are constructed by the tanh-coth method and the validity of the method has been tested.

Keywords- tanh-coth method ,solitary wave solution, soliton solution, travelling wave solution,burger-Fisher eqution.

1. INTRODUCTION

Most of the phenomena arise in science and engineering fields can be described by partial differential equations (PDEs). Two Dutch scientists Korteweg and de-Vries studied this concept of solitary waves and derived a nonlinear partial differential equation that admits solitary wave solutions and soliton solutions. The tanh-coth method is used by Abdul-Majid Wazwaz for handling several forms of nonlinear PDEs that admits travelling wave solutions.

The tanh-coth method :

Consider the nonlinear partial differential equation in general form as

$$P(v, v_x, v_t, v_{tt}, v_{xx}, v_{xxx}, \dots\dots..) = 0 \qquad (1)$$

Where $v = v(x,t)$is the solution of PDE (1). We use the wave transformation to transform $v(x,t)$ to $f(z)$ as .

$$v(x,t) = f(z) = f(x-ct)$$

Where $z = x - ct$

This gives

$$\frac{d}{dt} = -fc\frac{d}{dz},$$

$$\frac{d}{dx} = f\frac{d}{dz},$$

$$\frac{d^2}{dx^2} = f^2\frac{d^2}{dz^2},$$

$$\frac{d^3}{dx^3} = f^3\frac{d^3}{dz^3}$$

$$\frac{d^2}{dx^2} = f^2c^2\frac{d^2}{dz^2}$$

And so on

This wave variable $z = x - ct$ convert the PDE (1) into a ordinary differential equation of the form

$$R(v, v', v'', v''' \dots..) = 0 \qquad (2)$$

Then integrating equation (2) as long as all terms contain derivatives and in view of localized solution the integration of constants are considered to be zeros . However in some case the non-zero constants be used and handed as well. New finding of solitary ware solution to the equation (1) is equivalent to finding the solution of reduced ordinary differential equation(2).

In tanh method , tanh is used as new function or variable as all order derivatives of tanh function are represented by tanh function are represented by tanh function itself . for example ,we take

$$F = tanh(z)$$

$$F = sech^2(z)$$

$$= 1 - tanh^2(3)$$

$$= 1 - F^2$$

$$F'' = -2tanh(z)sech^2(z)$$

$$= -2tanh(z)\left(1 - tanh^2(z)\right)$$

$$= -2f(1 - F^2)$$

$$= -2F - 2F^3$$

Similarly $F''' = -2 + 8F^2 - 6F^4$

$$F^4 = 16F - 4F^3 + 24F^3$$

And so on

Now we introduce a new variable

$Y = tanh(\lambda z)$ OR $Y - coth(z)$ (3)

Where z is $x - ct$ and λ is wave number.

Then change of derivatives are :

$$\frac{d}{dz}(.) = (1 - Y^2)\frac{d}{dY}(.)$$

$$\frac{d^2}{dz^2}(.) = 2\lambda^2 Y(1 - Y^2)\frac{d}{dY}(.) + \lambda^2(1 - Y^2)^2\frac{d}{dY}(.)$$

$$\frac{d^3}{dz^3}(.) = 2\lambda^3 Y(1 - Y^2)(3Y^2 - 1)\frac{d}{dY}(.) + 6\lambda^2(1 - Y^2)^2\frac{d}{dY}(.) + \lambda^2(1 - Y^2)^3\frac{d^3}{dz^3}(.)$$

$$\frac{d^4}{dz^4}(.) = -8\lambda^4 Y(1 - Y^2)(3Y^2 - 1)\frac{d}{dY}(.) + 4\lambda^2(1 - Y^2)(9Y^2 - 2)\frac{d^2}{dY^2}(.) + 12\lambda^2(1 - Y^2)^3\frac{d^3}{dz^3}(.) + \lambda^4(1 - Y^2)^4\frac{d^2}{dY^2}(.) \quad (4)$$

Them this tanh-coth method allows us to use the finite expansion as substitution

$$Q(Y) = \sum_{m=0}^{N} a_m Y^m + \sum_{m=1}^{n} b_m Y^{-m} \quad (5)$$

Where Nis positive integer and in most of the cases it will be determined by balance method .

If we take $b_m = 0$ $m = 1$ to N, then (5) reduces to the standard original tanh method .That is in tanh method admits the use of finite expansion in the form

$$v(\lambda z) = Q(Y) = \sum_{m=0}^{N} a_m Y^m \quad (6)$$

All the remaining procedure in tanh method is similar as the procedure in tanh-coth method .

Then putting (5) into the reduced ordinary differential equation $(ODE)(2)$, gives an algebraic equation in integral powers of Y.

To carry out the balance method, form equation $(4) and (5)$, we note that the highest exponents for the function and its derivatives are given as follows.

$$v \to N$$

$$v^2 \to 3N$$

$$v^3 \to 3N$$

$$v^r \to rN$$

$$v' \to N + 1 \quad (7)$$

$$v'' \to N + 2$$

$$v''' \to N + 3$$

$$v^{(r)} \to N + r$$

$$(v')^2 \to (N + 1)^2$$

$$(v'')^2 \to (N + 1)^2$$

$$(v^r)^2 \to (N + r)^2$$

$$(v')^r \to (N + 1)^r$$

For determination of N, usually the balance method can be applied. That is balance the linear terms of highest order in the resulting algebraic equation in powers of Y with the highest order nonlinear term by using the above scheme (7) . Then collect all coefficients of power of Y i.e Y^0, Y^1, Y^2, Y^3……..in the resulting algebraic equation in Y and equate with zero .This will give a system of algebraic equations determine in parameter in d_m, b_m, c and λ.Form these equations determine the values of parameter c,λ and $a_m(m = 0toN)b_m(m = 0toN)$. Then by substituting the different sets of $a_m and b_m$ into equation (5)[into (6) in case of tanh method], we obtain an analytic solution $v(x,t)$in closed forms of different types.

The Burger-Fisher Equation

Consider the Burger-Fisher equation in the from

$$v_t - v_{xx} + vv_x - v(1-v) = 0 \qquad (8)$$

This equation describes the propagation of weak shock wave in a fluid and also the variation in vehicle density in highway traffic. This is one of the basic model equation that in fluid mechanics, unlike the kdv equation that combines the dispersion equation v_{xxx} effect with the nonlinear effect vv_x,in the Burger-Fishers equation combines the nonlinear effect vv_x with the dissipation vv_x effect. The Burger-Fisher equation used to describe traffic flow, the structures of shock wave and acoustic transmission .The Burger-Fishers equation is completely integrable and has different forms of travelling wave solutions.

By substitution

$$v(x,t) = f(z), \quad z = x - ct$$

The PDE (8) converted into ODE

$$-cv' - +2vv' + v'' - v(1-v) = 0 \qquad (9)$$

Balancing the nonlinear term v^2 , that has exponent 2N , with the highest order derivative term v'', that has exponent $N+2$, we have

$$2N = N + 2$$

So that

$$N = 2$$

The tanh-coth method suggests the substitution

$$Y = tan(\lambda z), \quad z = x - ct$$

And

$$v(x,t) = \sum_{m=0}^{2} a^m Y^m + \sum_{m=1}^{2} b^m Y^{-m}$$

$$= a_0 + a_1 Y + a_2 Y^2 + b_1 Y^{-1} + b_2 Y^{-2} \qquad (10)$$

Substituting (10) into equation (9) and proceeding as above, we find various following set of solutions.

$$(i)\ a_0 = -\frac{1}{4} \quad , a_1 = -\frac{1}{2} \ , b_1 = 0, a_2 = \frac{1}{4}, b_2 = 0, c = \frac{5}{\sqrt{6}} \ , \lambda = \frac{1}{2\sqrt{6}}$$

$$(i)\ a_0 = \frac{3}{8} \quad , a_1 = \frac{-1}{4} \ , b_1 = -\frac{1}{4}, a_2 = \frac{1}{16}, b_2 = \frac{1}{16}, c = \frac{5}{\sqrt{6}} \ , \lambda = \frac{1}{4\sqrt{6}}$$

The first sets gives the following kink solution

$$v_1(x,t) = \frac{1}{4}\left\{1 - tanh\left(\frac{1}{2\sqrt{6}}\left(x - \frac{5}{\sqrt{6}}t\right)\right)\right\}^2$$

The second set gives the following travelling wave solution

$$v_2(x,t) = \frac{1}{4}\left\{1 - coth\left(\frac{1}{2\sqrt{6}}\left(x - \frac{5}{\sqrt{6}}t\right)\right)\right\}^2$$

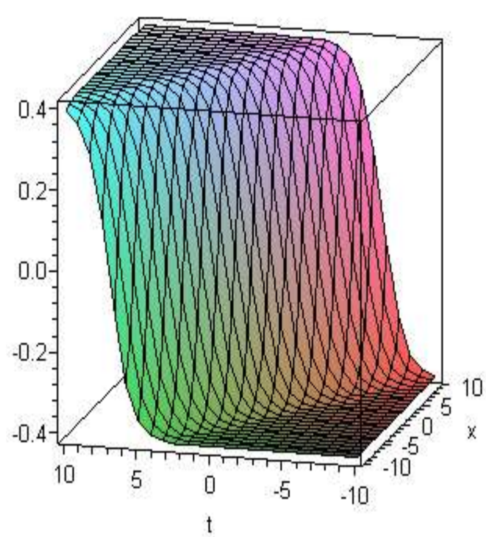

Fig.1.1: Time evolution graph of kink solution $v_1(x,t)$ in (4.119) with$-10 \le x,t \le 10$

CONCLUSION

The method is powerful mathematical technique for finding the travelling wave and solitary wave solutions of highly nonlinear partial differential equations. As a result we establish the exact solutions. The tanh-coth method is independent of the integrability of non-linear PDEs, so that it can be used to solve both types of integrable or non-integrable PDEs.

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