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Design of Stable PID Controllers for Time Delay System with Maximum Sensitivity Limitations

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Abstract— **In this paper, a method for designing PID controller is simulated and proposed for time delay system. For controller design, higher order system is approximated into first order plus delay time (FOPDT) model using frequency response method. The tuning parameters of the controller are obtained from maximum sensitivity value. It is considerably faster and simpler to determine. The stability region verification is done with dual locus diagram. The approach presented works well without sweeping over the parameters, and do not need complex numerical calculations. It determines search band of variables effectively for designing of advanced PID controllers using optimization method. For showing effectiveness of proposed controller, it is successfully designed and implemented for illustrative example in simulation study and obtained satisfactory results.**

***Keywords*— PID Controller, Time-delay system, Dual-locus diagram**

1. INTRODUCTION

In automatic process control system proportional-integral-derivative (PID) controllers are significantly and most commonly used in industries. In this, more than 90% of all control loops are PI/PID types [1]. The main reasons of the PID being used are its advantages like simple structure, high reliability, well understood control action and simple to implement. Many researchers in the past, developed number of PID design and tuning methods using first-order-plus-delay-time (FOPDT). Some of them are Ziegler- Nichols method [2], Cohen- Coon method [3], constant open loop transfer function method [4], synthesis method [5], internal model controller [6], and so on. However, these tuning methods have certain limitations, and often do not provide good tuning parameterizations of the PID controllers for high order- plus-delay-time (HOPDT) processes. The gain and phase margin (GPM) specifications method has been found many applications for designing of PI/PD/PID controllers for time delay and integral plus time delay processes [7,8,21]. In this method damping factor of the system is relating with phase margin of the systems and served as a measure of robustness. In GPM the solutions are normally obtained by numerically or graphically by means of trial-and-error, generally using the Bode plots. This method is certainly not suitable for systems having infinite phase crossover frequencies (e.g. systems without time delays and having number of zeros less than the number of poles by one i.e. system is not perfect). The main drawback of the GPM method is that the transfer function of the controlled systems is restricted to the FOPDT or SOPDT.

In this paper, PID controller design method is proposed for time-delay system based on reduced FOPDT model. The proposed method is based on the observation for a fixed value of K_d, the resulting set of stable PID controllers form a polyhedral sets in the three dimensional parameter space (K_p, K_i, K_d) and it is considerably faster and simpler to determine such regions. The stability region verification is done with dual locus diagram.

The organization of the paper is as follows: In Section 2 higher order system is approximated into FOPDT model based on two frequency points and design of PID Controller based on Stability Boundary are explained in detail. Section 3 presents, Dual Locus Diagram of closed loop system. Obtained simulation results are depicted in Section 4 followed by conclusions in Section 5.

2. MODEL ORDER APPROXIMATION AND COMPUTATION OF STABILITY BOUNDARY

Let us consider the general unity feedback control system as shown in Fig.1. In which, $G_p(s)$ is the plant with time delay time and $G_c(s)$ is controller which is to be design. These can be presented in

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the form as following.

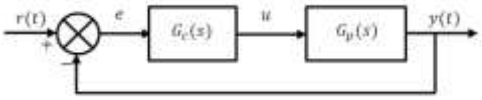

Fig. 1 Closed loop control Systems.

$$G_p(s) = G(s)e^{-t_0 s} = \frac{k}{\tau s + 1} e^{(-t_0 s)} \quad (1)$$

$$G_c(s) = K_p + {K_i}/_{s} + K_d s \quad (2)$$

The three unknown parameters in Eq. (1) namely k, t_0 and τ are steady state gain, delay time and time constant respectively and required to be determined. The higher order model $G'(s)$ is reduced int FOPDT model by fitting the frequency response at $\omega_1 = 0$ and $\omega_2 = \omega_c$ where ω_c is the phase crossover frequency of original system $G'(s)$. [15]

$$\begin{gathered} G_p(0) = G'(0) \\ |G_p(j\omega_c)| = |G'(\omega_c)| \\ \angle\{G_p(j\omega_c)\} = \angle\{G'(\omega_c) = -\pi\} \end{gathered} \quad (3)$$

$$\begin{gathered} k = G'(0) \\ \tau = \frac{\sqrt{\left(\frac{G'(0)}{|G'(j\omega_c)|}\right)^2 - 1}}{\omega_c} \\ t_0 = \frac{\pi - \tan^{-1}(\tau\omega_c)}{\omega_c} \end{gathered} \quad (4)$$

The problem is to compute the parameters of stable PID controller in Eq.(2). We consider the closed-loop transfer function as

$$G_{cl}(s) = G_c(s)G_p(s)/1 + G_c(s)G_p(s) \quad (5)$$

Substitute Eq. (1) into Eq. (5) and denote $G(s) = N(s)/D(s)$, the closed-loop characteristic polynomial can be

$$\Delta(s) = sD(s) + \left(K_d s^2 + K_p s + K_i\right)N(s)e^{(-t_0 s)} \quad (6)$$

Then by putting $s = j\omega$, and decomposing the numerator and denominator polynomials of $G(s)$ into real and imaginary part, there exists

$$G(j\omega) = \frac{N_r(\omega) + j\omega N_i(\omega)}{D_r(\omega) + j\omega D_i(\omega)} \quad (7)$$

Dropping ω for simplicity and using Euler's identity for $e^{(-j\omega t_0)} = \cos(\omega t_0) - \sin(\omega t_0)$ into Eq.(6), characteristic polynomial can be expressed as

$$\Delta(j\omega) = R_\Delta(\omega) - jI_\Delta(\omega) \quad (8)$$

Where $R_\Delta(\omega) = \left(K_i N_r - \omega^2 K_d N_r - \omega^2 K_p N_i\right)\cos(t_0\omega) + \omega\left(K_i N_i + K_p N_r - \omega^2 K_d N_i\right)\sin(t_0\omega) - \omega^2 D_i$

$$I_\Delta(\omega) = \omega\left(K_i N_i + K_p N_r - \omega^2 K_d N_i\right)\cos(t_0\omega) + \left(\omega^2 K_d N_r + \omega^2 K_p N_i - K_i N_r\right)\sin(t_0\omega) + \omega D_r \quad (9)$$

By equating

$$R_\Delta(\omega) = 0, I_\Delta(\omega) = 0 \quad (10)$$

Real and Imaginary components with0, we have from Eq. (9)

$$\begin{gathered} K_p Q(\omega) + K_i R(\omega) + K_d M(\omega) = X(\omega) \\ K_p S(\omega) + K_i T(\omega) + K_d U(\omega) = Y(\omega) \end{gathered} \quad (11)$$

Where

$$\begin{aligned} Q(\omega) &= \omega N_r \sin(t_0\omega) - \omega^2 N_i \cos(t_0\omega) \\ R(\omega) &= N_r \cos(t_0\omega) + \omega N_i \sin(t_0\omega) \\ M(\omega) &= -\omega^2\left((N_r \cos(t_0\omega) + N_i \sin(t_0\omega))\right) \\ X(\omega) &= \omega^2 D_i \end{aligned} \quad (12)$$

$$\begin{aligned} S(\omega) &= \omega N_r \cos(t_0\omega) + \omega^2 N_i \sin(t_0\omega) \\ U(\omega) &= \omega N_i \cos(t_0\omega) - N_r \sin(t_0\omega) \\ N(\omega) &= -\omega^2\left((\omega N_i \cos(t_0\omega) - N_r \sin(t_0\omega))\right) \\ Y(\omega) &= -\omega D_i \end{aligned} \quad (13)$$

It is impossible to find a unique expression for the parameters K_p, K_i and K_d as a function of ω from Eq.(11) because number of equations is less than that of required one. In classical tuning, K_i usually varies with K_d and/or K_p. Therefore, we could assume

$$K_d = \frac{K_i}{n} \quad (14)$$

Substituting Eq. (14) into Eq. (11), and dropping ω for simplicity. We can express

$$K_p = \frac{(XU - YR)}{(QU - RS)} - \frac{(MU - NR)(YQ - XS)}{(QU - RS)[\,n(QU - RS) + (MS - NQ)]}$$

$$K_i = \frac{n(YQ - XS)}{n(QU - RS) + (MS - NQ)}$$

$$K_d = \frac{(YQ - XS)}{n(QU - RS) + (MS - NQ)} \quad (15)$$

Remark 1: According to system stability condition, the stability of a linear closed-loop system can be determined from the location of poles in the s-plane. That is, if all the roots of characteristic polynomial Eq.(6) are less than 0, the

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system will be stable. Obviously, $K_i = 0$ is the boundary obtained for $\omega = 0$ ($i.e. s = 0$)into Eq. (6) since it divides parametric spatial dimension into stable and unstable regions. Generally speaking, the stability analysis is discussed in the frequency range $\omega\epsilon(0, \omega_c)$ because of the operability of controllers, where, ω_c is the crossover frequency of plant. Since the phase of $G_p(j\omega)$ at $\omega = \omega_c$ is$-\pi$,, from Eq. (1) and Eq. (7), one can obtain

$$\tan^{-1}\left(\frac{\omega N_i}{N_r}\right) - \tan^{-1}\left(\frac{\omega D_i}{D_r}\right) - (t_0\omega) = -\pi \quad (16)$$

Taking the tangent for the first two terms, and noticing that the tangent of other two terms $\tan(t_0\omega - \pi) = \tan(t_0\omega)$ let

$$F(\omega) = \frac{\omega(N_iD_r - N_rD_i)}{N_rD_r + \omega^2 N_iD_i} \quad (17)$$

Then by plotting $F(\omega)$ and $\tan(t_0\omega)$versusω, and note that $F(\omega) = \tan(t_0\omega)$, finding the minimum positive value of ω at which two plot intersects with each other, which is ω_c. The minimum positive value of ω in Eq. (15) which make $K_i =$ 0will be denoted as ω_0. . Comparing the ω_0 toω_c, the smaller one will be considered as stability frequency ω_{sf}, by which it easy to determine the stability boundary of PID controller parameter in Eq. (15)

Remark 2: When$K_i = 0$, the feedforward path transfer function $G_c(s)G_p(s)$ has the nature just like plant$G_p(s)$. So this is the special case with assumption in Eq. (14) and it is easy to understand in this case $\omega_{sf} = \omega_c = \omega_0$.

3. DUAL LOCUS DIAGRAM OF THE CLOSED-LOOP SYSTEM

After obtaining the stability regions of PID controllers for time-delay systems, the next step is to verify the stability of closed-loop systems. One of easy ways is dual-locus diagram as shown in Fig.4, which is another expression of Nyquist diagram based on the following argument principle.

Lemma 1(argument principal [20]) suppose that a function f is monomorphic in a simply connected domain$D$. Suppose further that C is a Jordan curve in$D$, followed in the positive (anticlockwise) direction and that f has no poles or zeros on$C$. If Z and P denote the number of the zeros and poles, respectively, of f in the interior of C, counted with multiplicities, then the variation of the argument of $f(s)$ along the Jordan curve C is

$$var(arg f(s), C) = 2\pi(Z - P) \quad (18)$$

And the winding number of $f(s)$ round the point 0, i.e. the number of times $f(s)$ winds round the origins, is

$$n(f(s), 0) = (Z - P) \quad (19)$$

The characteristic equation of the closed-loop system is usually written in the form

$$1 + L(s) = 0 \quad (20)$$

Where $L(s)$ is open loop transfer function. Here, the closed contour C is the Nyquist contour. Eq. (20) can be rearranged as

$$L_1(s) = L_2(s) \quad (21)$$

Then, the dual-locus diagram with respect to $L_1(s)$ and $L_2(s)$ is obtained when s traverses the Nyquist contour. The argument of $L_1(s) = L_2(s)$ is the angle between the vector joining the corresponding points on loci $L_1(s)$and $L_2(s)$, and the positive real axis. Thus, the system is stable (i.e. $Z = 0$) if and only if the variation of the argument of $L_1(s) = L_2(s)$ is zero. Correspondingly, for the closed-loop system with time delay as the form of Eq. (6), one can obtain

$$H(s) = \left(K_ds^2 + K_ps + K_i\right)N(s)/D(s) = -e(t_0s) \quad (22)$$

The stable condition of system is: either the loci of $H(s)$ and $-e(t_0s)$ have no intersection or the loci of $H(s)$ arrives at

the point of intersection earlier than that of $-e(t_0s)$ if the two loci intersect.

There are some limitations to determine all stabilizing PID controllers. It is the sufficient but not necessary condition for the calculation of stability boundary.

The key point for the calculation of stability boundary is to find a unique expression for parameters K_p, K_i and K_d as a function of ω from Eq. (9). Actually, using dual-locus can set up interdependence instead of hypothesis. The first step is to determine theω_s, at which the locus of $H(s)$ intersects with the unity circle. From Eq. (22) we have

$$H(s) = G_c(s)G(s) \quad (23)$$

Let $s = j\omega$ substituting Eq. (2) and Eq. (7) into Eq. (23). it can be written as (by dropping ω for simplicity)

$$H(j\omega) = (A + j\omega B)/(-\omega^2 D_i + j\omega D_r) \quad (24)$$

Where

$$A = K_i N_r - K_d N_r \omega^2 - K_p N_i \omega^2$$
$$B = K_i N_i - K_d N_i \omega^2 + K_p N_{re} \quad (25)$$

If the locus of$H(j\omega)$ intersects with the locus of$-e(j\omega t_0$, that is$H(j\omega = 1)$, the ω_s can be solved from the following equation as the function of K_p, K_i and K_d

$$a\omega^6 + b\omega^4 + c\omega^2 + d = 0 \quad (26)$$

Where

$a = K_d^2 N_i^2$

$b = \left(K_d N_r + K_p N_i\right)^2 - D_i^2 - 2K_d N_i\left(K_i N_i + K_p N_r\right)$

$c = \left(K_i N_i + K_p N_r\right)^2 - D_r^2 - 2K_i N_r\left(K_d N_r + K_p N_i\right)$

$$d = K_i^2 N_r^2 \quad (27)$$

The phase angles $\theta_1$of $H(j\omega)$ and θ_2 for $-e(j\omega t_0)$ at ω_s are respectively

$$\theta_1 = arg[H(j\omega_s)] = T\left(K_p, K_i, K_d\right)$$
$$\theta_2 = arg[-e(j\omega t_0)] = t_0\omega_s + \pi \quad (28)$$

According to dual-locus diagram method, another stability boundary condition is

$$\theta_1 - \theta_2 = 0 \quad (29)$$

By combining Eq. (29) with Eq. (11), we have

$$K_p Q(\omega) + K_i R(\omega) + K_d M(\omega) = X(\omega)$$
$$K_p S(\omega) + K_i T(\omega) + K_d U(\omega) = Y(\omega)$$
$$\theta_1 - \theta_2 = 0 \quad (30)$$

Thus, the equations set determines all stability closed surface for all stabilizing PID controllers.

Remark 3: Theoretically, it is possible to find all stability ranges of PID controllers parameters, for time-delay systems using the combination of graphical method and dual-locus diagram. The symbolic function of MATLAB makes calculation easy. However, some points are still need to be considered:

1. There are no less than 4 roots for Eq. (26), in which how to choose the ω_s (i.e. the minimum positive root).
2. Analyzing the argument of $H(j\omega)$is needed because different phases are determined byω_s.
3. Without hypothesis of Eq.(14), it is necessary to compare ω_0 with ω_c to obtain the stability frequency ω_{sf}.

The maximum sensitivity M_s has a nice geometrical interpretation in the Nyquist diagram for robustness and stability. This term is defined as

$$M_s = max_\omega \left|\frac{1}{1 + L(j\omega)}\right| \quad (31)$$

$1/M_s$ is the shortest distance from the critical point -1 to the Nyquist curve of forward path transfer function. This introduces a direct interpretation as a robustness measure, because it informs how much the process can change without causing instability. As mentioned by [1], the typical values of $M_s$are in the range of 1.2 to 2. Larger values ofM_s give systems those are faster but less robust. It is also useful that the range to consider is not too large. In this paper for simulation example the maximum value of M_s is restricted to≤ 2 to insure the stability and robustness of the system.

4. SIMULATION EXAMPLE In this section an illustrative example is simulated in MATLAB7.11.0 to show the effectiveness of the proposed algorithm. The controllers designed by the proposed algorithm are compared with other tuning techniques such as numerical optimization approach for FOPDT and SOPDT PI/PID design by [15]

Illustrative Example: Consider the higher order process given in [11]

$$G'(s) = \frac{e^{-0.5s}}{(s+1)(s+5)^2} \quad (32)$$

The obtained FOPDT model using Eq. (4) is given by

$$G_p(s) = \frac{0.04e^{-0.7919s}}{1 + 1.2638s} \quad (33)$$

In terms of the method presented in section 2, the plant $G_p(s)$ is controlled by a PID controller. With assumption of Eq. (14), substituting n = 4, from Eq. (15)

$$K_p = \omega\sin(0.7919) - \cos(0.7919)$$
$$K_i = 4\omega[\sin(0.7919\omega) + \omega\cos(0.7919\omega)]/(\omega^2 + 4)$$
$$K_d = \omega[\sin(0.7919\omega) + \omega\cos(0.7919\omega)]/(\omega^2 + 4) \quad (34)$$

Using Eq. (17), it is easy to find the crossover frequencyω_C as well asω_0. The stability frequency $\omega_{sf} = 2.3850$ is determined as shown in Fig.2. Substituting ω from 0 to 2.3850 into

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Eq. (34), a spatial curve is plotted in space coordinates$\left(K_p, K_i, K_d\right)$. The closed stable spatial dimension is shown in Fig.3.

When designing an optimal PID controller for a specific time-delay system, it is very effective to determine search band for optimum results in this stable spatial dimension without sweeping over all possible regions. If it gives$G_c(s) = 50.93 + 33.75/s + 8.44s$, it is easy to test the stability of the time delay system using dual-locus mentioned in section 3. From Eq. (22)

$$H(s) = \frac{0.04(8.44s^2+50.93s+33.75)}{1.2638s^2+s} \quad (35)$$

The dual-locus is shown in Fig.4. Firstly, compute the minimum value of ωat which the locus of $H(s)$ intersects with the unity circle, denotedω_s. Substituting$s = j\omega$into Eq. (35), at$\omega_s = 1.77$. Thus, the phase angles$\theta_1$of $H(s)$ andθ_2

For $e(-t_0 s)$ at $\omega_s$are respectively

$$\theta_1 = \tan^{-1}\left(\frac{2.0372\omega_s}{1.35 - 0.3376\omega_s}\right) + \tan^{-1}\left(\frac{1}{1.2638\omega_s}\right) + \pi = 5.0518$$

$$\theta_2 = 0.7919\omega_s + \pi = 4.5432 \quad (36)$$

Obviously, $\theta_1$is greater thanθ_2. According to the stability condition of dual-locus diagram, the locus $H(s)$ arrives at the intersection earlier than that of$-e(t_0 s)$. So this PID controller makes the time-delay system stable.

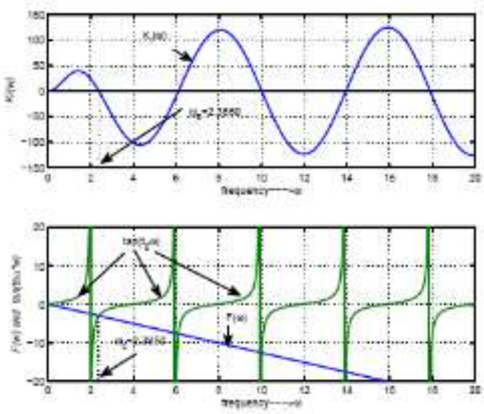

Fig. 2 Computation of ω_c *and* ω_0

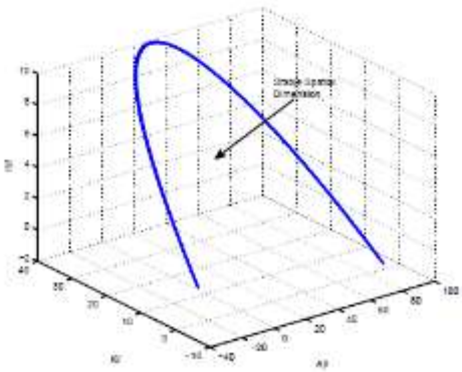

Fig. 3 3-Dimensional Region

The closed loop response of the system with PID controller designed by the proposed method is simulated and compared with the controller designed using Numerical optimization approach given by [15] as shown in Fig.5. for unit set point change at time $t = 0$.

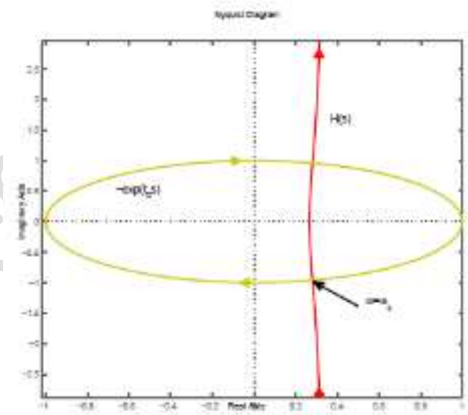

Fig. 4 Dual Locus Diagram

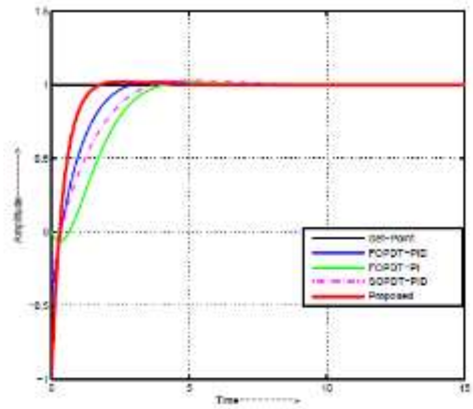

Fig. 5 Closed-loop Unit Step Response

Table I. Summary of Simulation Result $*OS$=% Overshoot, ST =Settling Time

Method	**Controller**			$*\boldsymbol{OS}$	$\cdot\boldsymbol{ST}$
	K_p	K_i	K_d		

Proposed	50.93	33.75	15.44	2.11	1.49
SOPDT-PID	23.85	18.33	8.22	3.10	5.76
FOPDT-PID	29.73	20.61	9.25	1.72	2.90
FOPDT-PI	16.11	13.10	-	0.87	4.03

IV. CONCLUSION

In this paper, an effective method of PID controller tuning using FOPDT model is demonstrated successfully. The effectiveness of the proposed method is based on stability regions of time-delay systems with PID controllers. With the assumption of proportional relation between K_i and K_d, a three dimensional parameter space (K_p, K_i, K_d) graph is plotted to show the stable spatial dimension.

The Dual-locus diagram shows the validity of the proposed method. The robustness constraint is integrated into the stable region determination algorithm, and the robustness is guaranteed by requiring the maximum sensitivity is less than a specified valueM_s. It is simple and advanced method, useful to determine the search band of variables of PID controller using optimization.

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