

Effect of Non uniform heat source / Sink for the Casson nanofluid in presence of Viscous Dissipation over the porous stretching sheet

¹Shravan B Kerur, ²Jagadish V Tawade*, ³Mahadev Biradar and ⁴Bharatkumar K. Manvi

^{1,4}Department of Mechanical Engineering, Basaveshvara Engineering College, Bagalkot-587102, Karnataka, INDIA

²Department of Mathematics, Bheemanna Khnadre Institute of Technology, Bhalki-585328, Karnataka, INDIA

³Department of Mathematics, Basaveshvara Engineering College, Bagalkot-587102, Karnataka, INDIA

Abstract

In this paper we have investigated the effects of boundary layer flow and heat transfer of a Casson nanofluids with non uniform heat source/sink and viscous dissipation over a permeable stretching sheet. The governing boundary layer equations of the flow are transformed into a set of non linear ordinary differential equations by using suitable similarity transformations. These equations are solved numerically by using Runge-Kutta method with efficient Shooting technique. The effects of various parameters like Casson parameter, porosity parameter, Eckert number, Non-uniform heat source/ sink parameter, Prandtl number, skin friction coefficient and Nusselt number on the flow field are studied and obtained numerical results are tabulated and presented with the aid of graphs. The obtained result reveals that with an increase in the Casson parameter, the velocity field decreases whereas the temperature profile increases.

Keywords: Casson parameter, nanofluids, porous medium, viscous dissipation, Non-uniform heat source/sink.

1. Introduction

Nanotechnology plays an important role in wide range of domains like, electronic devices, automobiles, aerospace, defence industry, biomedical science, solar water heating and cooling applications, power generation, wind and water turbines and many mechanical manufacturing industries. Crane[1] was the first to study the boundary layer flow past a stretching plate. He gave an exact solution for the originating problem. Later on, the boundary layer flow over linear and non-linear stretching sheet has pulled a great deal of interest of many of the researchers like Cortell[2], Bhattacharyya et.al[3], Mukhopadhyay [4], Rashidi and Pour[5] and Pal[6] etc.

Choi [7] was the first who employed that if nanometersized particles are suspended in base-fluid enhanced the heat transfer rate. M K Nayak et al. [8] investigated the magnetohydrodynamic nanofluid flow subject to porous matrix and convective heating past a permeable linear stretching sheet. In addition, the influences of velocity slip, viscous dissipation, Joule heating and non-linear thermal radiation are considered. The flow of Falkner-Skan equation over a wedge in the presence of slip condition was numerically discussed by Turkyilmazoglu [9]. K.P. Goyal et al. [10] studied the heat and mass transfer in a saturated porous wedge with impermeable boundaries.

Abel et al. [11] examined the effect of viscous dissipation on the MHD flow and heat transfer in a liquid film due to a stretching surface. To nanofluid and nano particle migration, Chamkha[12] discussed the impact of heat transport in a Power-law liquid along a wedge using finite difference scheme, numerically. The heat transfer aspects of non-Newtonian liquid motion engendered by a nonlinearly stretched surface with respect to viscous dissipation were examined by Kishan and Kavitha[13]. Mixed convective Marangoni flow of Casson material in presence of Joule heating, viscous dissipation, inclined magnetic field and nonlinear solar radiation in a thermally stratified medium was examined by Hayat et al. [14]. Baag et al. [15] studied MHD flow analysis on a stretching sheet in a

porous medium using DTM-Pade' and Numerical Methods (shooting method), the influences of various interesting parameters like as magnetic parameter, permeability parameter, and the power index is discussed. Yousif et al. [16] investigated numerical simulation for a thin liquid sheet over unsteady stretching shoot by using homotopy perturbation technique which found that when increasing both Darcy number and the unsteadiness parameter will decrease the thickness of the thin liquid film.

The thermal radiation effect of magnetic field on nanofluids plays a dominant role for flows in space technology and engineering processes operating at high temperatures. Particularly in polymer industry, if the entire system involving the polymer extrusion process is placed in a thermally controlled environment then radiative influence is useful in controlling heat transfer processes. The thermal radiative effect in the system leads to a final product of desired quality. On the other hand, the radiative flows of an electrically conducting fluid are encountered in electrical power generation, astrophysical flows, solar power technology, space vehicle re-entry, nuclear engineering applications, and other industrial areas. The effect of radiation on MHD flow and heat transfer problems has become more significant in industries and technological processes [17]. Recently Cai et al. [18] developed fractal based approaches. It was expected that the nanofluids have a better thermal conductivity efficiency than the base fluid like water, ethylene glycol mixture etc. Ramana Reddy et al. [19] presented the comparative study Effect of cross diffusion on MHD non-Newtonian fluids flow past a stretching sheet with non-uniform heat source/sink. B. Mahanthesh et al. [20] carried out the research on Marangoni convection in Casson liquid flow due to an infinite disk with exponential space dependent heat source and cross-diffusion effects. Ghadikolaei et al. [21] mixed convection on MHD flow of casson nanofluid over a non-linearly permeable stretching sheet has been investigated and analyzed numerically. Vajravelu et al. [22] carried out a mathematical analysis of the effects of thermo physical properties on the thin film flow of an Ostwald-de waele fluid over a stretching surface in the presence of viscous dissipation. Siddiqui and Shankar[23] have investigated the magnetohydrodynamic boundary layer flow and heat transfer of a non-Newtonian behavior of Casson fluid model over a stretching surface through a porous medium and obtained the results numerically using the Keller box method.

In most of the studies either thermal radiation is considered or the viscous dissipation. Present work has extracted the results of heat transfer for various parameters with combining thermal radiation and viscous dissipation. We reproduced the excellent results compared to previous researches under some limiting cases. FazleMabood et al. [24], Sameh E. et al. [24] MHD denotes study of the dynamics of electrically conducting fluids. It establishes a coupling between the Navier-Stokes equations for fluid dynamics and Maxwell's equations for electromagnetism. The main concept behind the MHD is that magnetic fields can induce currents in a moving conductive fluid, which, in turn create forces on the fluid and influence on the magnetic field itself. B J Gireesha et al. [26] studied entropy generation and heat transport analysis of Casson fluid flow with viscous and Joule heating in an inclined porous micro channel. Poom Kumam and Wejdan Deebani [27] carried out research on radiative MHD Casson Nanofluid Flow with Activation energy and chemical reaction over past nonlinearly stretching surface through Entropy generation. Wubshet Ibrahim and G. C. Shit [28-29] investigated the MHD slip flow of upper-convected Maxwell nanofluid over a stretching sheet with chemical reaction and Entropy on Unsteady MHD Flow of Casson Nanofluid over a Stretching Vertical Plate with Thermal Radiation Effect. Asim Aziz et al. [30] also investigated the entropy generation in MHD Maxwell nanofluid flow with variable thermal conductivity, thermal radiation, slip conditions, and heat source. Asia Yasmin [31] conducted the comprehensive investigation of mass and heat transfer in magneto hydrodynamics (MHD) flow of an electrically conducting non-Newtonian micropolar fluid because of curved stretching sheet. A. Vijayalakshmi [32] hydromagnetic pulsatile flow of a nanofluid between two parallel walls with thermal radiation is investigated. Water is considered as base fluid and Ag, CuO, Al₂O₃, TiO₂ are considered as nano particles. Brinkman and Maxwell Garnetts models are considered. Perturbation technique is employed to obtain the solution of the governing system. Challa Kalyan Kumar [33] investigated the magnetohydrodynamic pulsatile flow of Casson nanofluid through a vertical channel embedded in porous medium with thermal radiation and heat generation or absorption has been analysed using Buongiorno model. The influence of viscous and Joules dissipations are analysed. The aim of this study is to investigate the effects of Non uniform

heat source / sink and viscous dissipation on steady flow with heat transfer of the Casson nanofluid past a stretching sheet.

2. Formulation of the problem

Consider the steady two-dimensional flow of a Casson fluid near the stagnation-point on a heated stretching surface coinciding with the plane $y = 0$, the flow being confined to $y > 0$, where y is the coordinate normal to the surface. Two equal and opposite forces are applied along the x -axis (measured along the surface) so that the surface is stretched keeping the origin fixed. The rheological equation of state for an isotropic and incompressible flow of the Casson nanofluid is defined as follows (Ref.[23]),

$$\tau_{ij} = \begin{cases} 2 \left(\mu_B + \frac{P_y}{\sqrt{2\pi}} \right) e_{ij}, & \pi > \pi_c \\ 2 \left(\mu_B + \frac{P_y}{\sqrt{2\pi_c}} \right) e_{ij}, & \pi < \pi_c \end{cases}$$

$\pi = e_{ij}e_{ij}$, and e_{ij} is the $(i, j)^{th}$ component of deformation rate, n is the product of deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μ_B is the plastic dynamic viscosity of the non-Newtonian fluid, π product of the component of deformation rate with itself, π_c is critical value of the product and P_y is the yield stress of the fluid. The governing boundary layer equations of the flow are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu_{nf} \left(1 + \frac{1}{\gamma} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\nu_{nf}}{k_0} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{\rho C_p} \left(1 + \frac{1}{\gamma} \right) \left(\frac{\partial u}{\partial y} \right)^2 + \frac{q'''}{\rho C_p} \quad (3)$$

where u and v are the velocity components in the x and y directions respectively. ν_{nf} is the kinematic viscosity, ρ_f is the density of base fluid, $\gamma = \mu_B \frac{\sqrt{2\pi_c}}{P_y}$ is the non-Newtonian (Casson) parameter, σ is the electrical conductivity of the fluid, C_p is the specific heat, α_{nf} is the thermal diffusivity of the fluid, T is the temperature and k_0 is the permeability of the porous medium.

The non-uniform heat source/sink q''' from equation (3) is modeled as

$$q''' = \frac{k u_w(x)}{xv} [A^* (T_s - T_0) f' + (T - T_0) B^*], \quad (4)$$

Where A^* and B^* are the coefficients of space and temperature dependent heat source/sink respectively. Here we make a note that the case $A^* > 0$, $B^* > 0$ corresponds to internal heat generation and that $A^* < 0$, $B^* < 0$ corresponds to internal heat absorption. Further it is assumed that the induced magnetic field is negligibly small.

The governing equations are subject to the boundary conditions

$$\begin{aligned} u &= u_w(x) = bx, \quad v = 0, \\ T &= T_w = T_\infty + A \left(\frac{x}{l} \right)^2 \quad \text{at} \quad y = 0 \\ u &\rightarrow 0, \quad T \rightarrow T_\infty \quad \text{as} \quad y \rightarrow \infty \end{aligned} \quad (5)$$

Where $u_w = bx$, $b > 0$ is the stretching sheet velocity and A is a constant, x and y are Cartesian coordinates measured along stretching surface, l is the characteristic length, T is the temperature of the fluid, T_w is the temperature at the stretching surface (shear stress) and T_∞ is the temperature of the fluid far away from the stretching surface (free stream temperature),

We introduce the following suitable similarity variables

$$\psi = x\sqrt{bv} f(\eta), \quad \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \quad \eta = \sqrt{\frac{b}{\nu_f}} y \quad (6)$$

Where ψ is the stream function which is defined in the usual way as

$$u = \frac{\partial\psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial\psi}{\partial x}$$

Where η is the dimensionless similarity variable, θ is the dimensionless temperature,

Substituting (6) in equations (2) and (3), the set of ordinary differential equations results in

$$\left(1 + \frac{1}{\gamma}\right) f''' + \phi_1 (f f'' - f'^2) - k f' = 0 \quad (7)$$

$$\theta'' + \left(\frac{k_f}{k_{nf}} Pr \phi_2\right) f \theta' + \left(1 + \frac{1}{\gamma}\right) Ec f''^2 - (A^* f' + B^* \theta) = 0 \quad (8)$$

And the conditions in (5) becomes

$$f = 0, \quad f' = 1, \quad \theta = 1 \quad \text{at} \quad \eta \rightarrow 0$$

$$f' \rightarrow 0, \quad \theta \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty \quad (9)$$

Here γ is the Casson parameter, $k = \frac{\nu_f}{k_0 b}$ is the porosity Parameter, ϕ is the nanoparticle volume

fraction (where $\phi_1 = \frac{\nu_f}{\nu_{nf}}$, and $\phi_2 = \frac{\rho_{nf}}{\rho_f}$), $Pr = \frac{\nu_f}{\alpha_f}$ is the Prandtl number and $Ec = \frac{u_w}{C_p [T_w - T_\infty]}$ is the constant

Eckert number, and the prime denotes differentiation with respect to η .

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x , which are defined as

$$C_f = \frac{\tau_w}{\rho_f u_w^2} \quad \text{and} \quad Nu_x = \frac{x q_w}{k(T_w - T_\infty)} \quad (10)$$

Where τ_w is the skin friction or the shear stress along the stretching surface and

q_w is the heat flux from the surface, which are given by

$$\tau_w = \left(\mu_B + \frac{P_y}{\sqrt{2\pi c}}\right) \left(\frac{\partial u}{\partial y}\right)_{y=0} \quad \text{and} \quad q_w = -k \left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (11)$$

Substituting the transformations in (6), (10) and (11), we obtain

$$Re_x^{\frac{1}{2}} C_f = \left(1 + \frac{1}{\gamma}\right) f''(0), \quad \text{and} \quad Re_x^{-\frac{1}{2}} Nu_x = -\theta'(0) \quad (12)$$

Where $Re_x = \frac{u_w x}{\nu}$ is the local Reynolds number.

3. Numerical solution

The coupled ordinary differential equations (7) and (8) are highly non-linear with the appropriate boundary conditions (9) are solved numerically by the efficient fourth order Runge-Kutta algorithm along with the numerical shooting technique. Set of these nonlinear ordinary differential equations are of third order in f second order in θ reduced into a system of simultaneous ordinary equations as shown below,

$$\frac{df_0}{d\eta} = f_1, \quad \frac{df_1}{d\eta} = f_2, \quad \left(1 + \frac{1}{\gamma}\right) \frac{df_2}{d\eta} = \phi_1 (f_0 f_2 - f_1^2) - k f_1 \quad (13)$$

$$\frac{d\theta_0}{d\eta} = \theta_1, \quad \frac{d\theta_1}{d\eta} = -\left(\frac{k_f}{k_{nf}} Pr \phi_2 \theta_1 f_0\right) - (A^* f_0 + B^* \theta_0) - Ec \left(1 + \frac{1}{\gamma}\right) f_2^2 \quad (14)$$

The associated Boundary conditions take the form,

$$f_0(0) = 0, \quad f_1(0) = 1, \quad \theta_0(0) = 1 \quad (15)$$

$$f_1(\infty) = 0, \quad \theta_0(\infty) = 0, \quad (16)$$

Here $f_0(\eta) = f(\eta)$ and $\theta_0(\eta) = \theta(\eta)$. This requires the initial values $f_2(0)$ and $\theta_1(0)$ and hence suitable guess values are chosen and later integration is performed. A step size of $\Delta\eta = 0.001$ is chosen with an error of tolerance 10^{-6} . In order to solve this system of equations using Runge-Kutta method with shooting technique. In order to get the desired values one should need three more missing initial conditions. However, the values of $f'(\eta)$ and $\theta(\eta)$ are known when $\eta \rightarrow \infty$, these end conditions are used to obtain an unknown initial conditions at $\eta = 0$ by using suitable shooting technique.

4. Analysis of the Result

The study of Casson nanofluid flow with non uniform heat source/sink and viscous dissipation in porous medium is investigated. The numerical calculation has been carried out to compute the velocity profiles, temperature profiles, skin friction coefficient and local Nusselt number for various values of the parameters that describe the flow characteristics. The determination of this section is to examine the physical consequences of different embedding parameters on the velocity and temperature profiles which are illustrated in figures [1–9].

Figures [1-2] reveals the effect of Casson parameter γ and porosity parameter k on the velocity profile $f'(\eta)$. Figure 1 describes that the fluid velocity falls for higher values of Casson parameter γ due to the inverse relation of γ with yield stress leads to the fact that increasing values of γ decrease the yield stress, that is, an increase in the Casson parameter reduces fluids plasticity. A similar result was observed by the authors [23] for considered fluid flow. In Figure 2 reveals that an increasing values of porosity parameter k declares that the fluid velocity sharply in the vicinity of stretching sheet. Physically, this shows that fluid velocity adjacent to the sheet is less than the velocity of normal stretching sheet.

Figures [3–9] reveals that the effect of γ , k , Pr , Ec , ϕ_2 , A^* and B^* on temperature profiles of Casson nanofluid over the horizontal stretching sheet. Figure 3 depicts the effect of Casson nanofluid parameter γ on the temperature profile $\theta(\eta)$. It is noticed that the temperature of the nanofluid enhances with the increasing values of Casson nanofluid parameter γ and hence the thermal boundary layer thickness increases as the elasticity stress parameter γ is increased. Figure 4 represents the effects of porosity parameter k on the temperature profile, and it is noticed that with an increasing values of porosity parameter, the temperature of the fluid increases. Figure 5 reveals that the fluid temperature increases as the values of nanoparticle's volume fraction ϕ_2 increases.

Figure 6 depicts the effect of increasing the Prandtl number on the fluid temperature distribution. An increase in the Prandtl number as expected is seen to reduce the fluid temperature above the sheet. This is because as the Prandtl number increases, the thermal boundary layer becomes thinner. Thus the rate of thermal diffusion drops, resulting in the fluid temperature dropping as well.

In Figure 7 we display the effect of the Eckert number on the temperature profiles. An increase in the values of the Eckert number is seen to increase the temperature of the fluid at any point above the sheet. Increasing the Eckert number allows energy to be stored in the fluid region as a consequence of dissipation due to viscosity and elastic deformation.

Figures [8 & 9] illustrates that an increasing values of non-uniform heat source/sink parameters A^* and B^* increases the temperature distribution in the boundary region. Generally, positive values of the non-uniform heat source/sink parameter acts as heat generators and negative values of the non-uniform heat source/sink parameter acts as heat absorption from the boundary layer. Figure 10 displays that for increase in nanoparticle's volume fraction ϕ , skin friction coefficient grow up where as local Nusselt number in figure [11 & 12] also shows the same effect for an increase in the Prandtl number Pr and Eckert number Ec .

5. Concluding remarks

In this study we have analyzed the velocity and heat transferred characteristics of two dimensional flow based on Casson nanofluid over a porous stretching sheet with the effects of non-uniform heat source/sink. Various non dimensional governing parameters that influence the velocity distribution and heat transfer rate which are discussed and presented with the aid of graphs. The following effects are observed.

- An increase in the Casson parameter eventually decreases the velocity profiles.
- The skin friction $-f''(0)$ increases with an increase in nanoparticle volume fraction ϕ and the porosity parameter k .
- The Nusselt number increases with increasing values of the Prandtl number.
- The presence of non uniform heat source/sink helps in the development of temperature profile. For positive values of the non-uniform heat source/sink parameter acts as heat generators and negative values of the non-uniform heat source/sink parameter acts as heat absorption from the boundary layer.

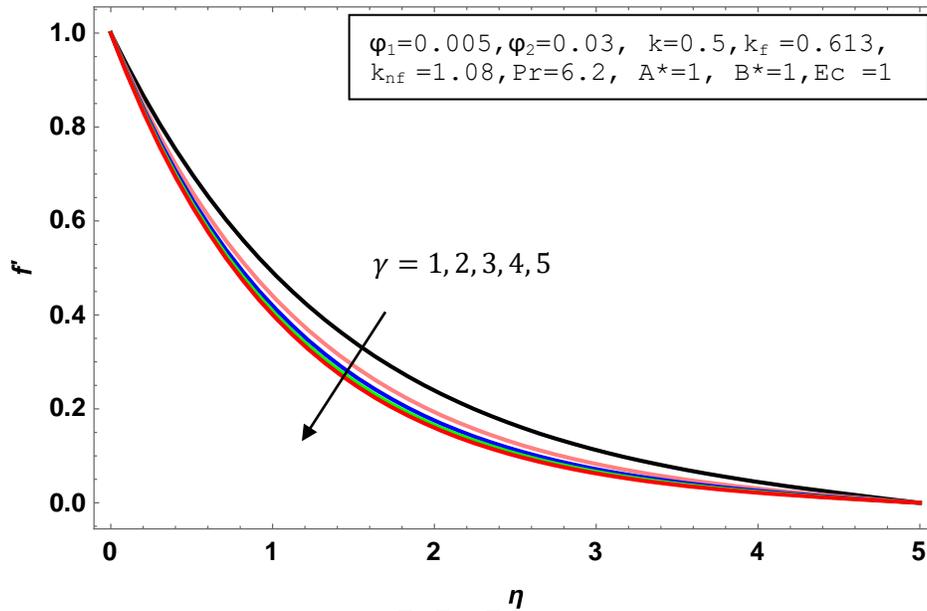


Figure.1. Velocity profile $f'(\eta)$ for various values of Casson parameter γ

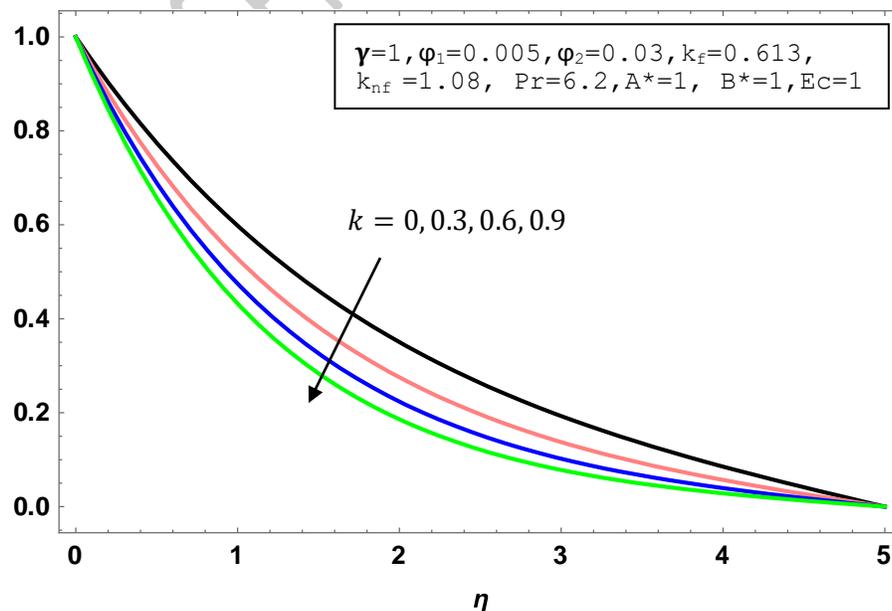


Figure.2. Velocity profile $f'(\eta)$ for various values of Porosity parameter k

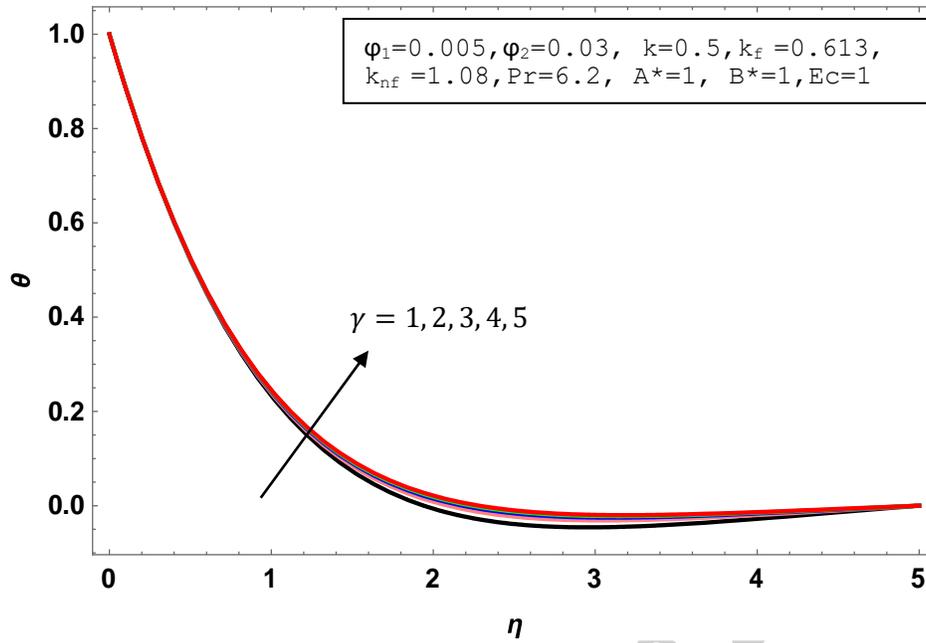


Figure.3. Temperature profile $\theta(\eta)$ for various values of Casson parameter γ

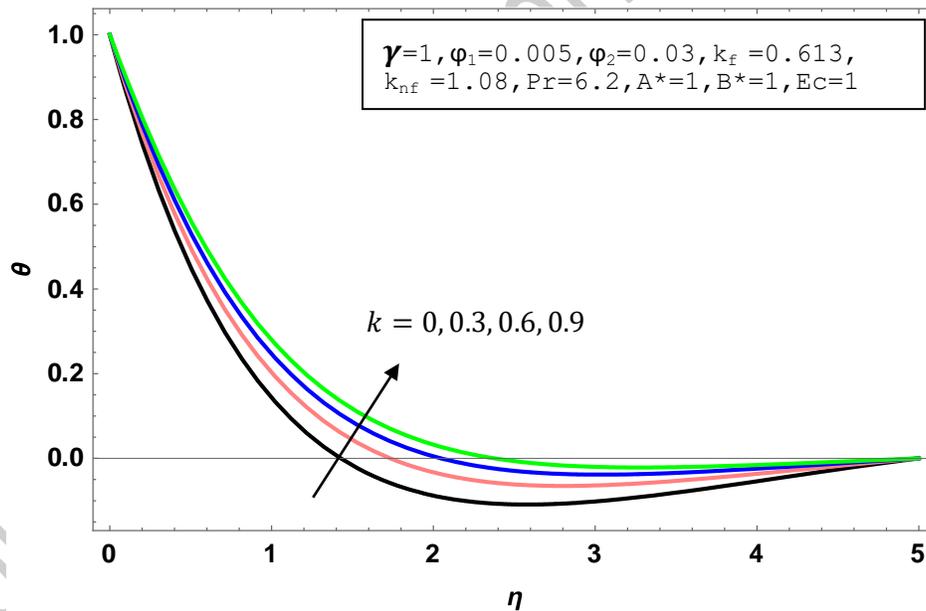


Figure.4. Temperature profile $\theta(\eta)$ for various values of Porosity parameter k

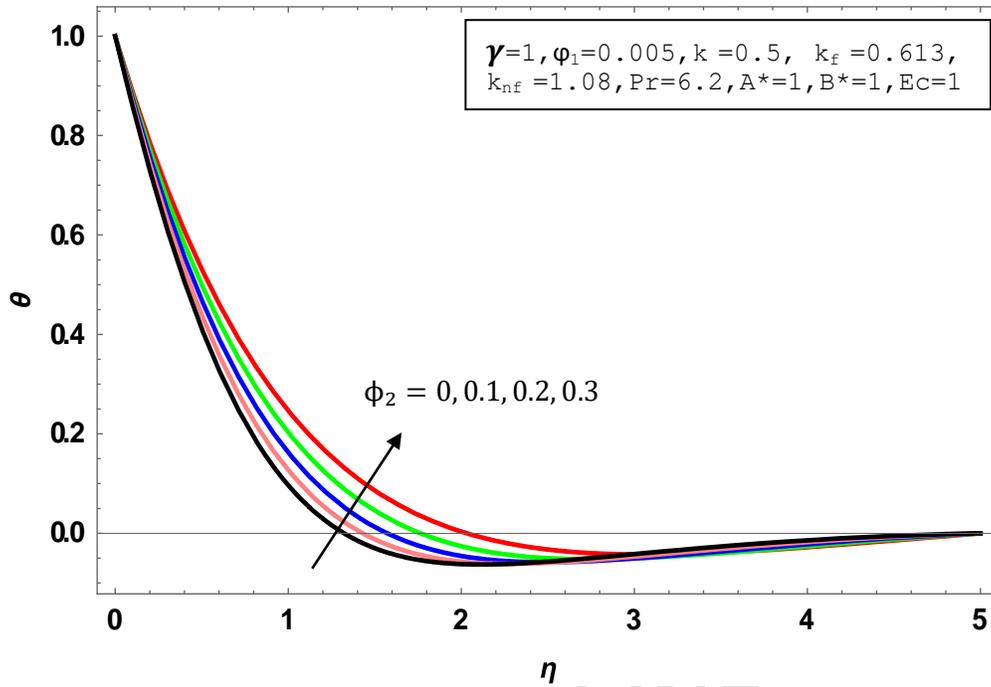


Figure.5. Temperature profile $\theta(\eta)$ for various values of nanoparticle parameter ϕ_3

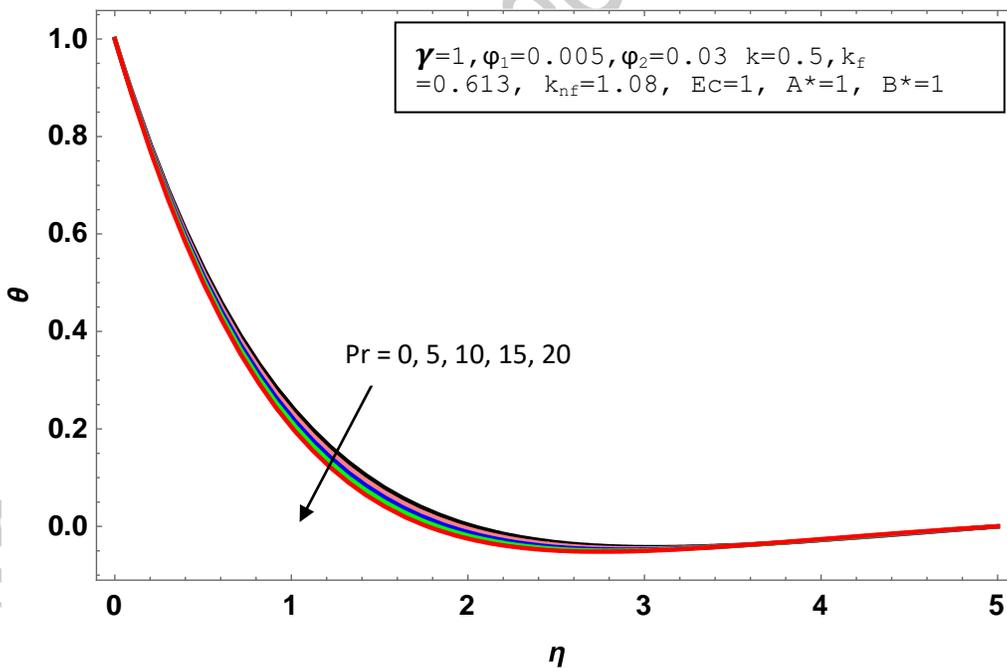


Figure.6. Temperature profile $\theta(\eta)$ for various values of Prandtl number Pr

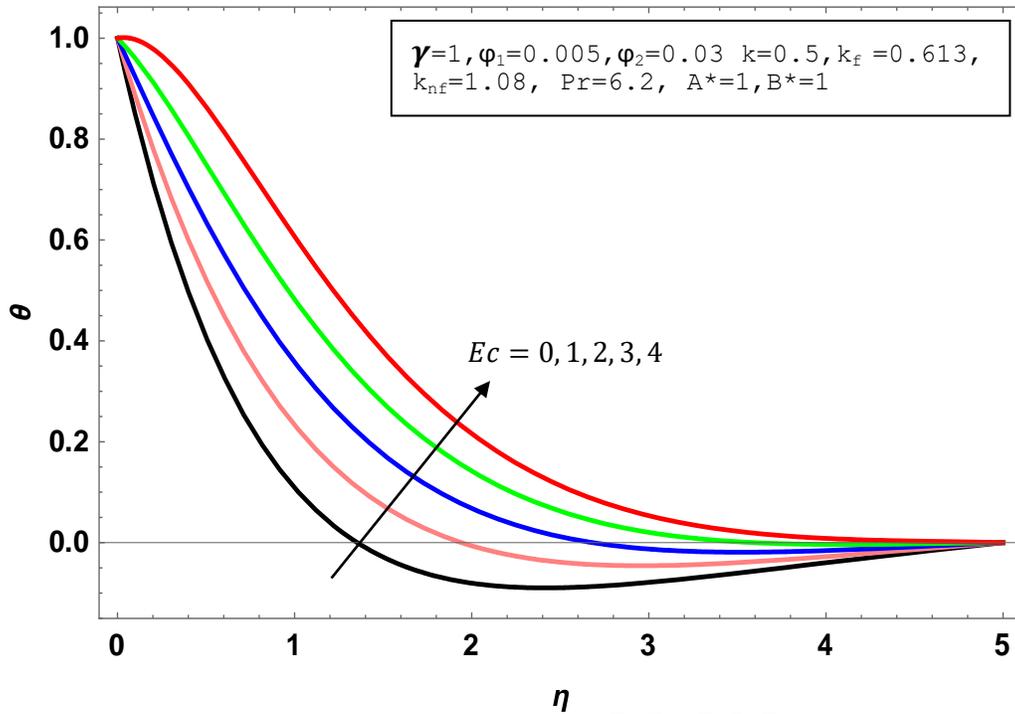


Figure.7. Temperature profile $\theta(\eta)$ for various values of Eckert number Ec

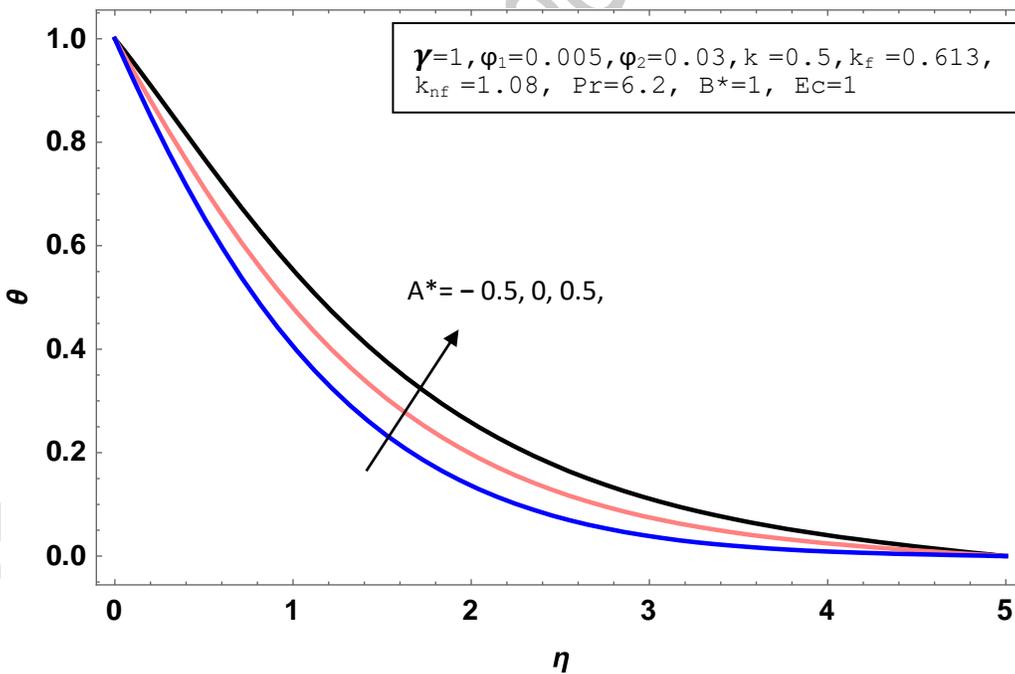


Figure.8. Velocity profile $\theta(\eta)$ for various values of A^*

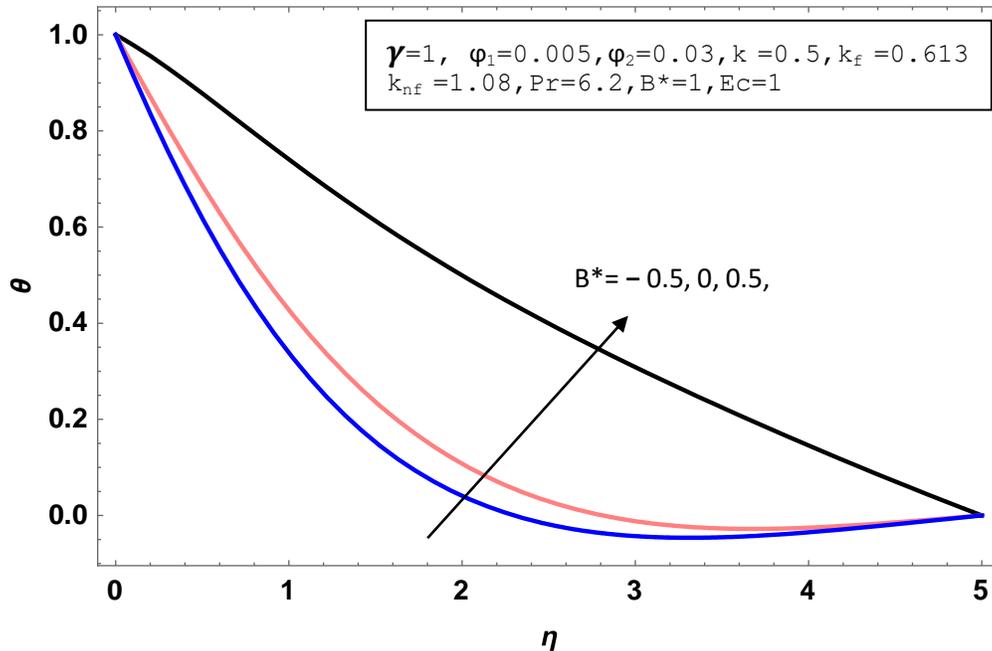


Figure.9. Temperature profile $\theta(\eta)$ for various values of B^*

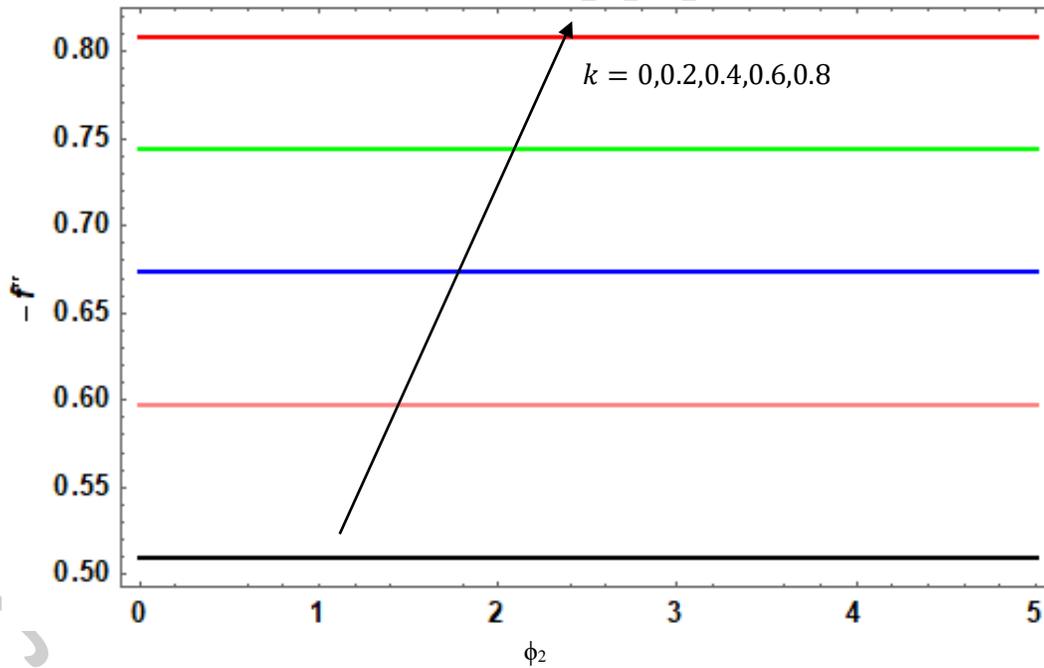


Figure.10. Skin friction $-f''(0)$ with nanoparticle volume fraction ϕ_3 for various values of porosity parameter k

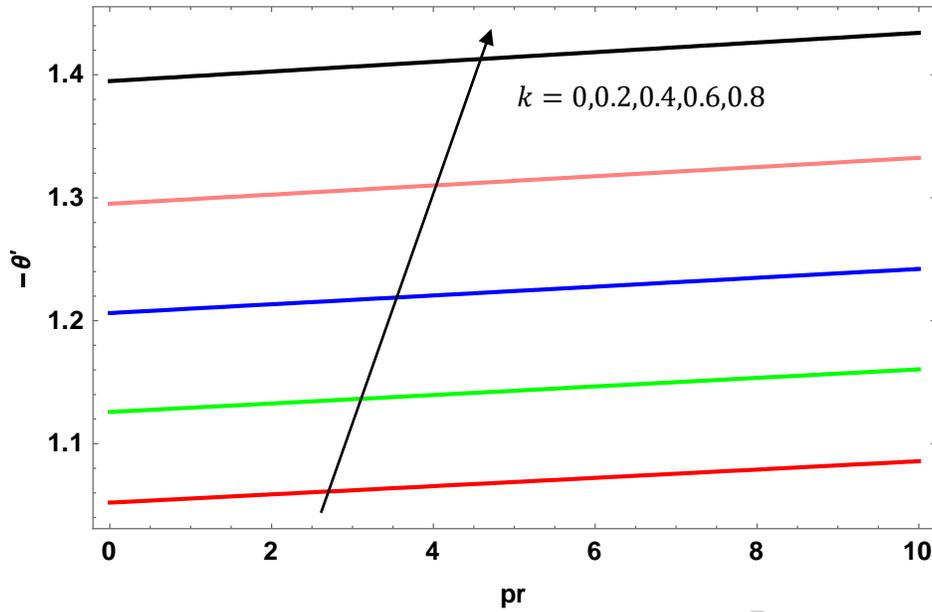


Figure.11. Variation of heat transfer coefficient $-\theta'(0)$ with Prandtl number Pr for various values of Porosity parameter k .

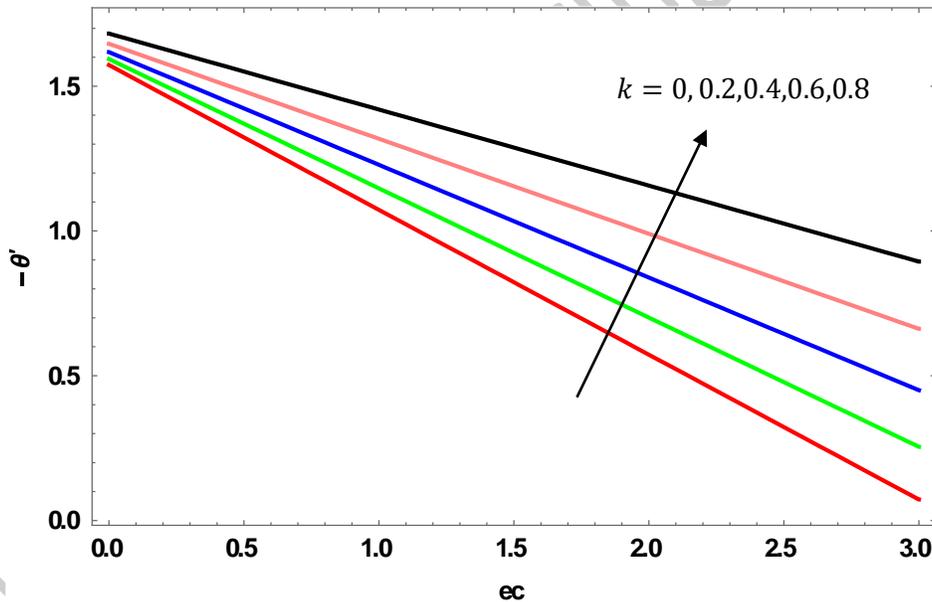


Figure.12. Variation of heat transfer coefficient $-\theta'(0)$ with Eckert number Ec for various values of Porosity parameter k .

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