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Non-uniform heat source/sink and thermal radiation effects on liquid film flow of Casson fluid thin film over a Stretching Sheet

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Abstract

In the present study, the flow of thin uniform Casson liquid film on an unsteady stretching sheet is studied numerically. By using suitable similarity transformations the governing non-linear partial differential equations have been transformed into non-linear ordinary differential equations. The efficient numerical method with shooting technique is used to solve the system of non-linear ordinary differential equations which describes liquid thin film flow of Casson fluid and heat transfer over a stretching sheet in presence of unsteadiness parameter, Prandtl number, Eckert number, non uniform heat source/sink, thermal radiation, and viscous dissipation. From the results we observe that the film thickness enhances with the Casson parameter in the boundary layer of the flow.

Key words: Casson liquid, thin film, viscous dissipation, Non uniform heat source/sink, Stretching sheet.

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1. Introduction

Fluid thin films can be seen in a variety of situations in nature and in various industrial applications, such as membrane in biophysics, tear films in eyes, coating flows, microfluidic engineering, etc. The ubiquitous presence of thin films in nature and technology has led the scientists and researchers to address the mechanism involved in the flow. Despite the diversity of phenomena and applications, the mathematical model is quite similar if the fluid is sufficiently viscous. The flow caused by stretching boundary arises frequently in materials manufactured by extrusion, glass-fiber and paper production. Following the pioneering work of Crane [1], recently the flow and heat transfer characteristics past a stretching/shrinking sheet with and without slip condition at the surface were studied by many authors [2-6]. On the other hand, Wang [7] was the first who discussed the steady flow of a viscous fluid outside of a stretching hollow cylinder by considering the ambient fluid at rest.

The characteristics and analysis of flow and heat transfer of thin films have attracted the attention of many researchers due to their abundant applications in the last two decades. This refers to their multiple applications in engineering such as food stuff processing, reactor fluidization, wire and fiber coating, cooling of metallic plates, drawing of a polymer sheet, aerodynamic extrusion of plastic sheets, continuous casting, rolling, annealing and thinning of copper wires. In the extrusion process, this understanding is crucial for maintenance of the surface quality of extrudate. All coating process requires a smooth glossy surface for the best product appearance and properties like as low friction, strength and transparency. As the quality of product in the extrusion processes depends considerably on the film flow and heat transfer characteristics of a thin liquid over a stretching plate, investigated and analysis of momentum and heat transfer in such processes is essential. The study of flow characteristics in a liquid thin film over an unsteady stretching sheet is studied by Wang [8] and this study is extended by Dandapat et al. [9] by including heat transfer characteristics. It is reported that the fluids having more friction slower the velocity generates considerable amount of heat, for instance, in the case of extrusion of plastic sheets, and thus rate of heat transfer may change appreciably due to viscous dissipation. Sarma and Rao [10] have investigated heat transfer in a viscoelastic fluid over a stretching sheet analytically in the presence of viscous dissipation and internal heat generation.

Sarojamma et al. [11] presented mathematical model of MHD unsteady flow induced by a stretching surface embedded in a rotating Casson fluid with thermal radiation magnetohydrodynamics. Abel et al. [12] examined the effect of non uniform heat source on MHD heat transfer in a liquid film over an unsteady stretching sheet numerically with viscous dissipation. More recently the authors like Megahed [13], Kalyani et al [14], Vijaya et al. [15], Eldabe et al [16] etc… have examined the impact of flow on the heat transfer of a Casson fluid in thin film on a stretching sheet. In the present study we have examined the flow of Casson liquid film fluid with heat transmission and having the impact of thermal radiation and non uniform heat source/sink over a time dependent stretching surface. Most of the aforementioned studies have neglected the combined effect of thermal radiation and non-uniform heat source/sink on the heat transfer which is important in view point of desired properties of the outcome. In the present study we include the same for heat transfer analysis in a Casson thin liquid film from an unsteady stretching sheet. The obtained results in comparison with that of Wang[7], Abel et al [12], Megahed [13], Kalyani et al [14], without any doubt, from the tables, we can claim that our results are in excellent agreement with that of aforementioned results under some limiting cases. Some of the relevant analysis on the subject are due to author's [17-20].

Most of the above-mentioned studies have ignored the combined effects of viscous dissipations, thermal radiation and non uniform heat source/sink on the liquid film flows of Casson fluid over stretching sheet. The considered flow is due to stretching of the sheet from a slot through two equal and opposite forces. Thus, the intention behind the present investigation is to understand the liquid thin film flow behavior of a Casson fluid thin film over a stretching sheet in presence of viscous dissipation, thermal radiation and non uniform heat source/sink. Although for such flows, the richness of ideas and phenomena discussed in the proposed study can be expected to lead to highly productive interactions across disciplines.

2. Formulation of the problem

Consider a steady, laminar, two-dimensional boundary layer flow and heat transfer of an incompressible fluid combined with non uniform heat source and thermal radiation over a stretching sheet. The sheet coincides with the plane $y = 0$ and the flow is confined to $y > 0$. The flow is generated due to linear stretching of the sheet caused by the simultaneous applications of two equal and opposite forces along the x–axis as shown in the Figure 1.

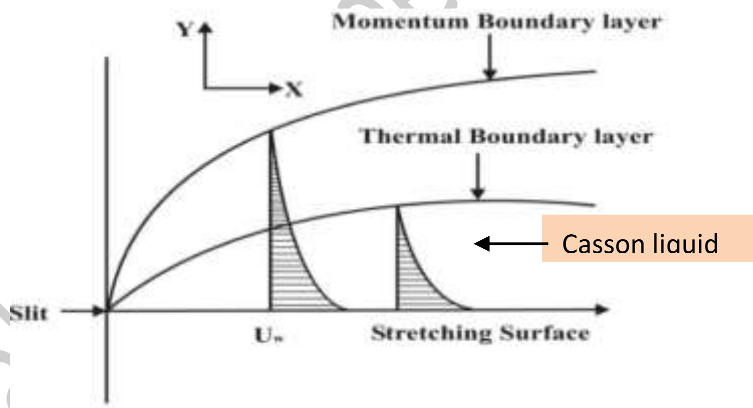

Figure 1: Schematic representation of the flow diagram.

The continuous sheet is parallel to x-axis and moves in its own plane with a velocity

$$U(x,t) = \frac{bx}{1-\alpha t} \tag{1}$$

where α and b are positive constants with dimension per time. The stretching sheet's temperature and concentration T_s is assumed to vary with the distance x from the slit as

$$T_s(x,t) = T_0 - T_{ref}\left[\frac{bx2}{2v}\right](1-\alpha t)^{-3/2} \tag{2}$$

The constitutive equation of the Casson fluid can be written as

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \frac{P_y}{\sqrt{2\pi}}\right)e_{ij}, & \pi > \pi_c \\ 2\left(\mu_B + \frac{P_y}{\sqrt{2\pi_c}}\right)e_{ij}, & \pi < \pi_c \end{cases} \tag{3}$$

where τ_{ij} is the $(i,\ j)^{th}$ component of the stress tensor, μ_B is the plastic dynamic viscosity of the non-Newtonian fluid, P_y is the yield stress of the fluid, π is the product of the component of deformation rate with itself, namely, $\pi = e_{ij}e_{ij}$, and e_{ij} is the $(i,\ j)^{th}$ component of deformation rate, and π_c is the critical value of π depends on non-Newtonian model. Under these assumptions, equations of the flow in the liquid film are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} \tag{5}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}\left(1 + \frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)^2 - \frac{4\sigma^*}{3\rho C_p k^*}\frac{\partial T^4}{\partial y} + \frac{q'''}{\rho C_p} \tag{6}$$

Where u and v are the velocity components of fluid in x- and y- directions, T is the temperature, μ is the dynamic viscosity, $\beta = \mu_B\frac{\sqrt{2\pi_c}}{P_y}$ is the Casson parameter, ρ is density, C_p is specific heat at constant pressure, k is the thermal conductivity, σ^* represents the Stefen Boltzman constant, k^* denoted the mean absorption coefficient.

Expanding T^4 by using Taylor's series about T_∞ and neglecting higher order terms, we have,

$$T^4 \cong 4{T_\infty}^3 T - 3{T_\infty}^4$$

Therefore $\frac{\partial}{\partial y}\left(-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}\right) = -\frac{16{T_\infty}^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial y^2}$

The non-uniform heat source/sink (see [12]) is modeled as

$$q''' = \frac{ku_w(x)}{x\upsilon}[A^*(T_s - T_0)f' + (T - T_0)B^*], \tag{7}$$

Where A* and B* are the coefficients of space and temperature dependent heat source/sink respectively. Here we make a note that the case $A^* > 0,\ B^* > 0$ corresponds to internal heat generation and that $A^* < 0,\ B^* < 0$ corresponds to internal heat absorption. Further it is assumed that the induced magnetic field is negligibly small. The corresponding boundary conditions of the flow problem are given by

$$u = U, \quad v = 0, \quad T = T_s \quad \text{at} \quad y = 0, \tag{8}$$

$$\frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = 0 \quad \text{at} \quad y = h, \tag{9}$$

$$v = \frac{dh}{dt} \quad \text{at} \quad y = h. \tag{10}$$

At this juncture we make a note that the mathematical problem is implicitly formulated only for $x \geq 0$. Further it is assumed that the surface of the planar liquid film is smooth so as to avoid the complications due to surface waves. The influence of interfacial shear due to the quiescent atmosphere, in other words the effect of surface tension is assumed to be negligible. The viscous shear stress $\tau = \mu\left(\frac{\partial u}{\partial y}\right)$ and the heat flux $q = -k\left(\frac{\partial T}{\partial y}\right)$ vanish at the adiabatic free surface (at y =h)

The following similarity transformations are introduced

$$\eta = \left[\frac{b}{v(1-\alpha t)}\right]^{\frac{1}{2}} y, \qquad \psi = x\left[\frac{bv}{1-\alpha t}\right]^{\frac{1}{2}} f(\eta), \tag{11}$$

$$T = T_o - T_{ref}\left[\frac{bx^2}{2v(1-\alpha t)^{\frac{3}{2}}}\right]\theta(\eta), \quad \theta(\eta) = \frac{T-T_0}{T_s-T_0} \tag{12}$$

Also $\psi(x, y)$ is the stream function and the velocity components can be obtained as

$$u = \frac{\partial\psi}{\partial y} = \frac{bx}{1-\alpha t} f'(\eta) \qquad \text{and} \qquad v = -\frac{\partial\psi}{\partial x} = -\left(\frac{bv}{1-\alpha t}\right)^{\frac{1}{2}} f(\eta) \tag{13}$$

Where prime denotes the differentiation with respect to η. The mathematical problem defined through equations (5) –(7) are transformed to the following non-linear boundary value problem as

$$\left(1+\frac{1}{\beta}\right) f''' + \left[ff'' - S\left(f' + \frac{\eta}{2}f''\right) - (f')^2\right] = 0 \tag{14}$$

$$\left(1+\frac{4}{3}Nr\right)\theta'' + Pr\left[f\theta' - 2f'\theta - \frac{S}{2}(\eta\theta' + 3\theta) + Ec\left(1+\frac{1}{\beta}\right)(f'')^2 + A^* f + B^*\theta\right] = 0$$
(15)

Subject to the boundary conditions

$$f'(0) = 1, \qquad f(0) = 0, \qquad \theta(0) = 1, \tag{16}$$

$$f''(\beta) = 0, \qquad \theta'(\beta) = 0, \tag{17}$$

$$f(\beta) = \frac{S\beta}{2}. \tag{18}$$

Here $S \equiv \frac{\alpha}{b}$ is the dimensionless unsteady parameter, $Pr = \frac{\rho C_p\, v}{k}$ is the Prandtl number, $Nr = \frac{4\sigma^* T_\infty{}^3}{k\,k^*}$ is the thermal radiation, $Ec = \frac{U^2}{C_p(T_s-T_0)}$ is the Eckert number. Further prime indicates differentiation with respect to η and β denotes the value of the similarity variable η at the free surface so that equation (11) gives

$$\beta = \left(\frac{b}{\upsilon(1-\alpha t)}\right)^{\frac{1}{2}} h. \tag{19}$$

Since β is an unknown constant, which should be determined as a whole from the set of the present boundary value problem. The rate of change of film thickness can be obtained as follows.

$$\frac{dh}{dt} = -\frac{\alpha\beta}{2}\left(\frac{\upsilon}{b(1-\alpha t)}\right)^{\frac{1}{2}}. \tag{20}$$

Thus the kinematic constraint at $y = h(t)$ given by equation (11) transforms into the free surface condition (20).
The surface drag coefficient C_{fx} and Nusselt number Nu_x which play a significant role in estimating the surface drag force, the rate of heat transfer

$$C_{fx}Re_x^{\frac{1}{2}} = -2\left(1+\frac{1}{\beta}\right)f''(0), \tag{21}$$

and $$Nu_x Re_x^{-\frac{1}{2}} = \theta'(0) \tag{22}$$

where $\mathrm{Re}_x = \frac{Ux}{\upsilon}$ is the local Reynolds number.

The coupled ordinary differential equations (14) and (15) are non-linear and exact analytical solutions are not possible. Equations (14) and (15) with the appropriate boundary conditions (16) – (18) are solved numerically by the efficient fourth order Runge-Kutta-Felberg algorithm along with the numerical shooting technique. These equations are converted into a set of first order differential equations as follows:

$$\frac{df_0}{d\eta} = f_1, \quad \frac{df_1}{d\eta} = f_2, \quad \left(1+\frac{1}{\beta}\right)\frac{df_2}{d\eta} = S\left(f_1 + \frac{\eta}{2}f_2\right) + f_1^{\,2} - f_0 f_2 \tag{23}$$

$$\frac{d\theta_0}{d\eta} = \theta_1, \ \left(1+\frac{4}{3}Nr\right)\frac{d\theta_1}{d\eta} = Pr\left(\frac{S}{2}(3\theta_0 + \eta\theta_1) + 2\theta_0 f_1 - \theta_1 f_0 - Ec\left(1+\frac{1}{\beta}\right)f_2^{\,2} - A^* f_0 - B^*\theta_0\right) \tag{24}$$

The associated Boundary conditions take the form,

$$f_0(0) = 0, \quad f_1(0) = 1, \quad \theta_0(0) = 1 \tag{25}$$
$$f_0(\beta) = \frac{S\beta}{2}, \quad f_2(\beta) = 0, \quad \theta_1(\beta) = 0 \tag{26}$$

Here $f_0(\eta) = f(\eta)$ and $\theta_0(\eta) = \theta(\eta)$. This requires the initial values $f_2(0)$ and $\theta_1(0)$ and hence suitable guess values are chosen and later integration is performed. A step size of $\Delta\eta = 0.01$ is chosen. The value of β is obtained in such a way that the boundary conditions $f_0(\beta) = \frac{S\beta}{2}$ is satisfied with an error of tolerance 10^{-8}.

3. Table of values

Table 1. Comparison of β and $f''(0)$ with the earlier published result when $\beta \to \infty$ for various values of S with an error of tolerance 10^{-8}

S	Wang[1]		Abel et al.[5]		Megahed [13]		Kalyani et all[14]		Present study	
	β	$-\frac{f''(0)}{\beta}$	β	$-f''(0)$	β	$-f''(0)$	β	$-f''(0)$	β	$-f''(0)$
0.4	5.122490	1.307785	4.981455	1.134098	4.981450	1.134096	4.981455	1.134098	4.981448	1.19677904
0.6	3.131250	1.195155	3.131710	1.195128	3.131710	1.195126	3.131710	1.195128	3.131710	1.21744369
0.8	2.151990	1.245795	2.151990	1.245805	2.151994	1.245806	2.151990	1.245805	2.151990	1.23784971
1.0	2.543620	1.277762	1.543617	1.277769	1.543616	1.277769	1.543617	1.277769	1.543616	1.25799643
1.2	1.127780	1.279177	1.127780	1.279171	1.127781	1.279172	1.127780	1.279171	1.127780	1.27788890
1.4	0.821032	1.233549	0.821033	1.233545	0.821032	1.233545	0.821033	1.233545	0.821033	1.29753250
1.6	0.576173	1.491137	0.576176	1.114937	0.576173	1.114938	0.576176	1.114937	0.576176	1.31693527
1.8	0.356383	0.867414	0.356390	0.867416	0.356389	0.867414	0.356390	0.867416	0.356390	1.33609566

Table 2. Variation of $\left(1+\frac{1}{\beta}\right)f''(0)$ and $-\theta'(0)$ for various values of S, and with Kalyani et al[14] with an error of tolerance 10^{-8}

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S	β	Kalyani et al [14]		Present results	
		$-\left(1+\frac{1}{\beta}\right)f''(0)$	$-\theta'(0)$	$-\left(1+\frac{1}{\beta}\right)f''(0)$	$-\theta'(0)$
0.8	0.5	2.47352	1.355718	2.50341110	0.47073412
1.0		2.504972	1.409314	2.59394902	0.56141482
1.2		2.478698	1.450178	2.68156399	0.63299901
1.4		2.365328	1.463985	2.76655680	0.69337388
0.5	1.0	1.577477	1.289227	2.04553892	0.47226902
	2.0	1.585218	1.276834	1.77172560	0.47292199
	3.0	1.591939	1.269177	1.67042993	0.47300349
	4.0	1.596655	1.264258	1.61740012	0.47299498

Table 3. Comparison of $-\theta'(0)$ for various values of Pr, Ec, Nr, A* and B*

Pr	Ec	Nr (Radiation parameter)	A*	B*	$-\theta'(0)$ Kalyani et al [14]	$-\theta'(0)$ Present results
0.7	1.0	0.5	0.0	0.0	1.108487	0.90460652
1.0					1.355718	1.11860071
2.0					1.982332	1.66981755
3.0					2.465460	2.09648677
1.0	0.0	0.5	0.0	0.0	1.315220	1.16438301
	1.0				0.910217	0.70655715
	2.0				0.505218	0.24873133
	3.0				0.100220	0.20909505
1.0	0.1	0.5	0.0	0.0	1.355718	1.11860055
		1.0			1.121379	0.91565736
		1.5			0.969474	0.78650627
		2.0			0.860352	0.69547968
1.0	1.0	0.5	−0.5	0.5	Neglected	1.10474585
			0.0		Neglected	1.10474585
			0.5		Neglected	1.10474585
			1.0		Neglected	1.10474585
1.0	1.0	0.5	0.5	−0.5	Neglected	1.10474585
				0.0	Neglected	1.10474585
				0.5	Neglected	1.10474585
				1.0	Neglected	1.10474585

4. Results and Discussion:

The system of nonlinear ordinary differential Equations (14) and (15) subjected to the boundary conditions (16-18) are solved numerically for the diverse values of physical parameters such as unsteadiness parameter S, thermal radiation Nr, Eckert number Ec, Prandtl number Pr, Casson parameter λ and Non uniform heat source/sink parameters A* and B*. Numerical results were achieved through Runge-Kutta method with shooting scheme detailed description of the method. For the validation of present numerical scheme, the results are presented in the tabular form (see tables 1,2 and 3) and examined with (Kalynani et al, Abel et.al and Megahed [13]) in some limiting case. The numerical values are in good agreement as displayed from Table.1 presents the effects of emerging parameters on the skin friction coefficient.

Figure 2(a) and 2 (b) reveals the effects of the film thickness β during the flow motion for a fixed values of unsteadiness parameter S = 0.8 and S = 1.2 respectively. It is observed that, increasing values in film thickness β decreases the flow velocity of the liquid film.

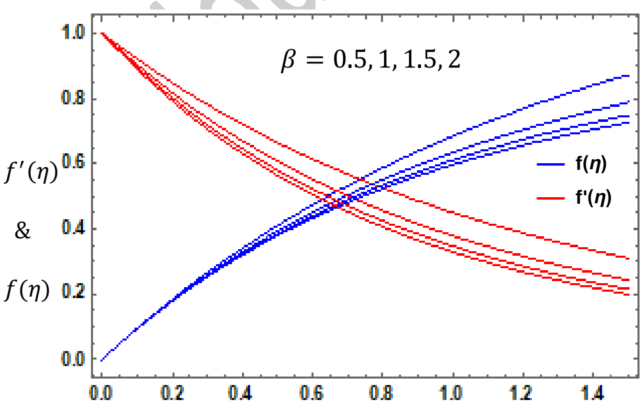
Figures 3(a) and 3(b) elucidates the influence of thermal radiation of Casson fluid on the temperature profile. From these figures we observe that the effect of thermal radiation enhances the temperature of the boundary layer. For large values of radiation parameter, generates a significant amount of heating to the Casson fluid which enhances the fluid temperature profile and thicker thermal boundary layer thickness.
The effects of Prandtl number Pr is a ratio of momentum diffusivity to thermal diffusivity. When Prandtl number Pr is high fluid has low thermal conductivity, which decreases the thermal boundary layer thickness and conduction. Accordingly, form Figures 4(a) and 4(b) it is seen that with increasing Prandtl number the thermal boundary layer and temperature profiles upsurge.
The influence of Eckert number Ec on the temperature profile is shown in figures 5(a) and 5(b). It reveals from the figure that temperature increases by increasing the values of Ec. Physically this behavior is observed because in presence of viscous dissipation heat energy is stored in the fluid and there is more significant generation of heat along the sheet.
Figures 6(a) and 6(b) represents the effects of Casson parameter λ on temperature profile. It is observed that the film thickness decreases for higher values of λ.
Figures 7(a) and 7(b) reveal for the temperature field for different values of A*. By analyzing the graphs it reveals that smaller values of A* decrease the temperature field in the boundary region.
Figure 8(a) and 8(b) shows the effect of temperature dependent heat source/sink parameter B* on the temperature profile $\theta(\eta)$. Temperature profile is decreasing function of B* when B*<0. Here energy is absorbed and in result the temperature significantly drops within the boundary layer.
Figure 9 depicts the change in the fluid velocity profile on the Casson fluid for various values of unsteadiness parameter S. It is observed from this figure that the fluid velocity increases with an increase in S also reveals the thinning of the film.
Figure 10 shows the temperature profiles on the Casson fluid for various values of unsteadiness parameter S. From this figure it is seen that the temperature profile decreases with increasing unsteadiness parameter S. This reveals that the rate of cooling is much faster for higher values of unsteadiness parameter S.

5. Conclusion

Effects of non-uniform heat source/sink and thermal radiation on liquid film flow of Casson fluid over a stretching sheet are examined. Main findings of the presented analysis are mentioned below.

- The unsteadiness parameter S increases the skin friction coefficient and the local Nusselt number.
- Increasing values of the non-uniform heat source or sink parameters enhances the thermal boundary layer thickness.
- Temperature and thermal boundary layer thickness increase when thermal radiation parameter Nr increases.
- Nusselt number decays considerably when A* and B* increase.

6. Results are shown with the aid of graphs as below:



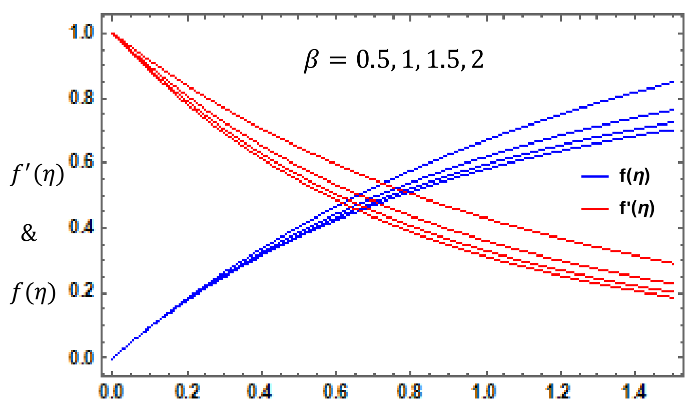


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Figure 2a: Variation of film thickness β for unsteadiness parameter S=0.8

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Figure 2b: Variation of film thickness β for unsteadiness parameter S=1.2

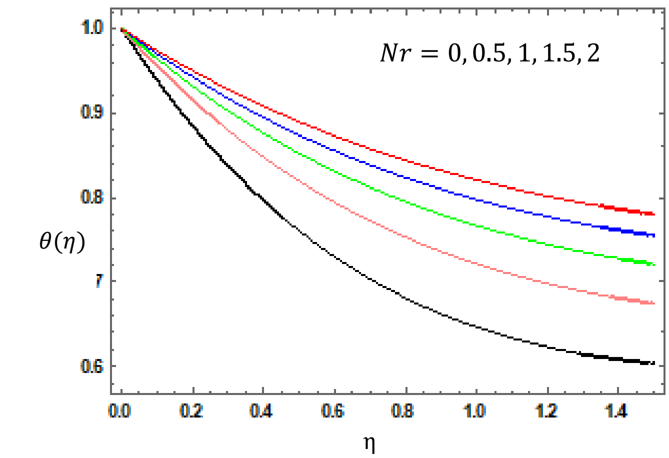


Figure 3a: Variations of thermal radiation parameter Nr on the temperature profile of Casson fluid for unsteadiness parameter S = 0.8

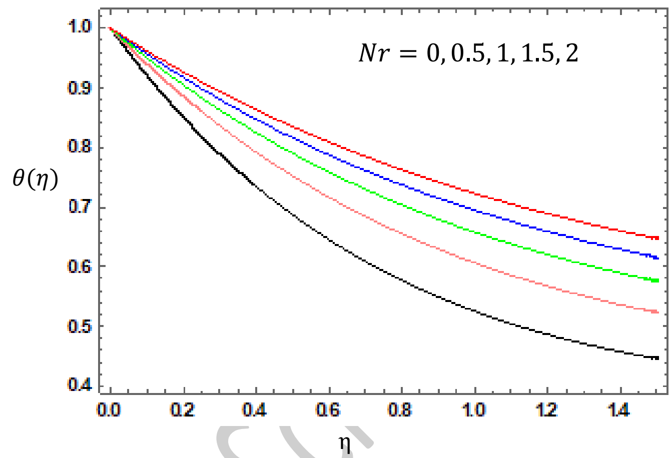


Figure 3b: Variations of thermal radiation parameter Nr on the temperature profile of Casson fluid for unsteadiness parameter S = 1.2

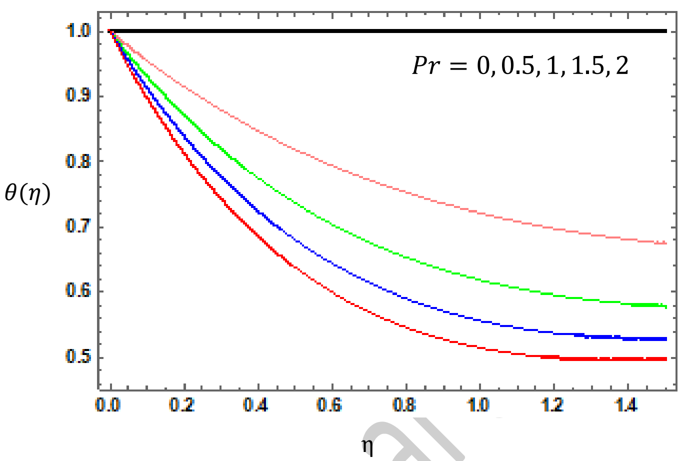


Figure 4a: Variations of Prandtl number Pr on the temperature profile of Casson fluid for unsteadiness parameter S = 0.8

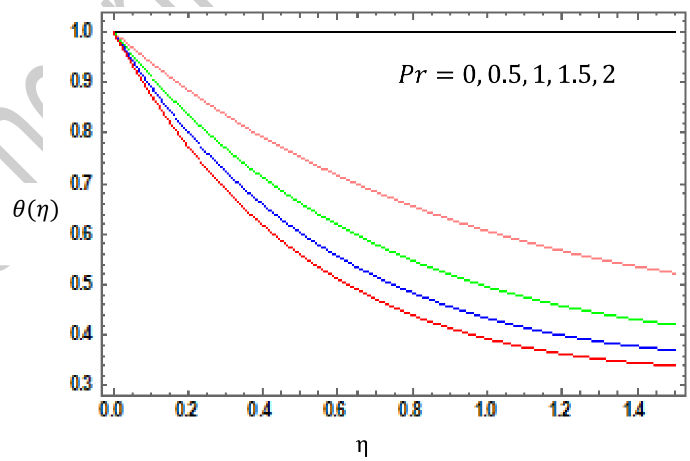
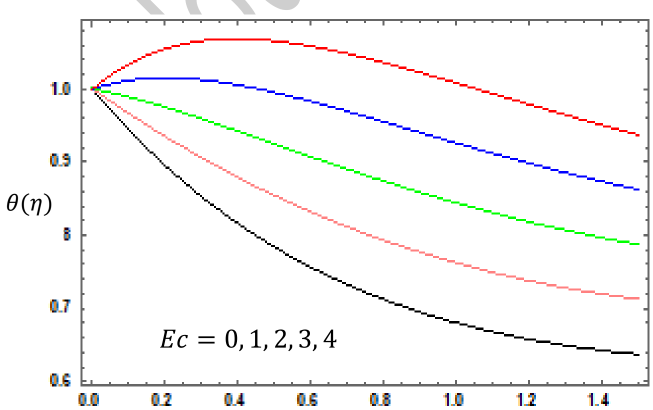


Figure 4b: Variations of Prandtl number Pr on the temperature profile of Casson fluid for unsteadiness parameter S = 1.2



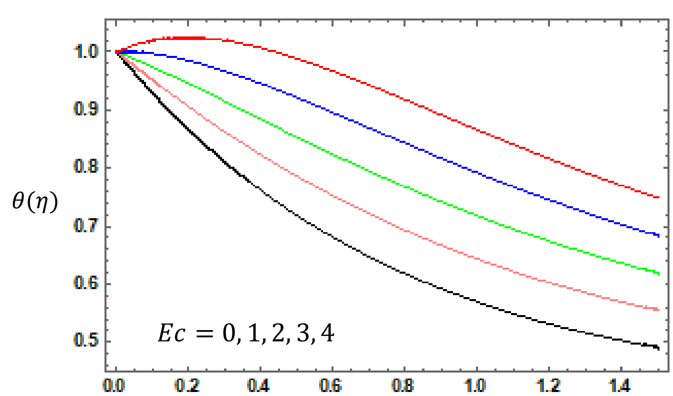


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Figure 5a: Variations of Eckert number Ec on the temperature profile of Casson fluid for unsteadiness parameter S = 0.8

η

Figure 5b: Variations of Eckert number Ec on the temperature profile of Casson fluid for unsteadiness parameter S = 1.2

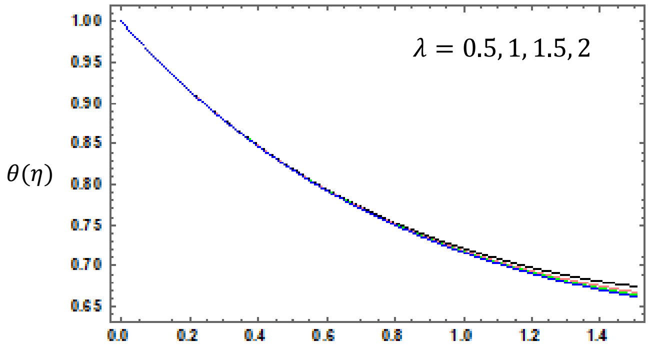

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Figure 6a: Variations of Casson parameter λ on the temperature profile for the unsteadiness parameter S = 0.8

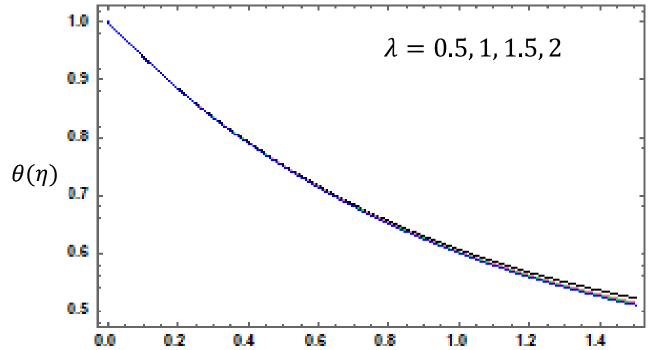

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Figure 6b: Variations of Casson parameter λ on the temperature profile for the unsteadiness parameter S = 1.2

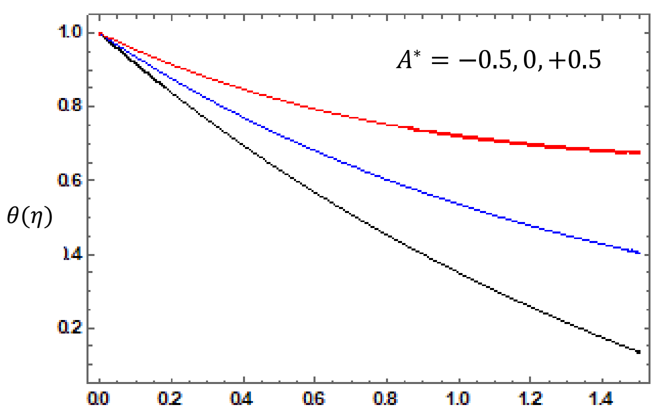

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Figure 7a: Variation of non-uniform heat source/ sink parameter A* on the temperature profile of Casson fluid for unsteadiness parameter S = 0.8

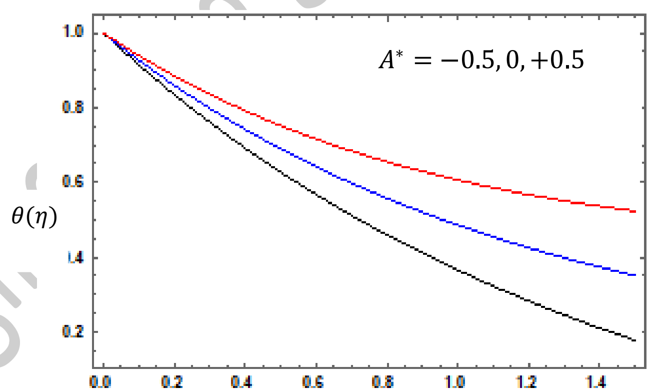

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Figure 7b: Variation of non-uniform heat source/ sink parameter A* on the temperature profile of Casson fluid for unsteadiness parameter S = 1.2

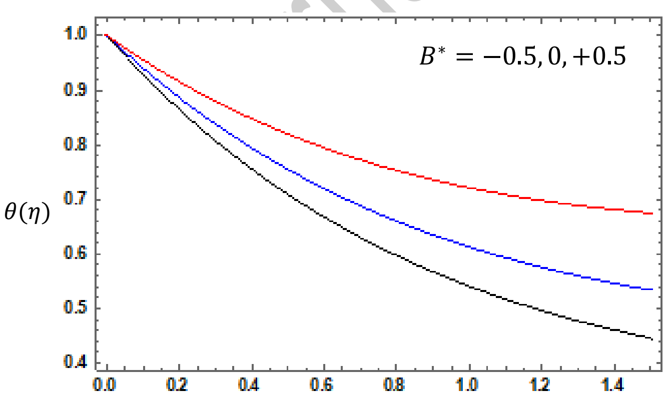

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Figure 8a: Variation of non-uniform heat source/ sink parameter B* on the temperature profile of Casson

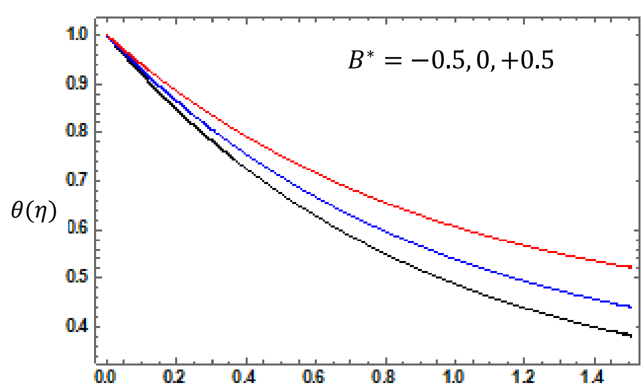

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Figure 8b: Variation of non-uniform heat source/ sink parameter B* on the temperature profile of Casson

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fluid for unsteadiness parameter S = 0.8 fluid for unsteadiness parameter S = 1.2

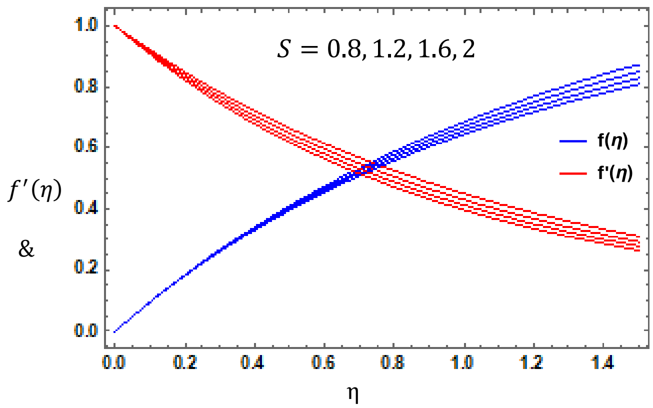

Figure 9 : Variation of unsteadiness parameter S on the velocity distribution

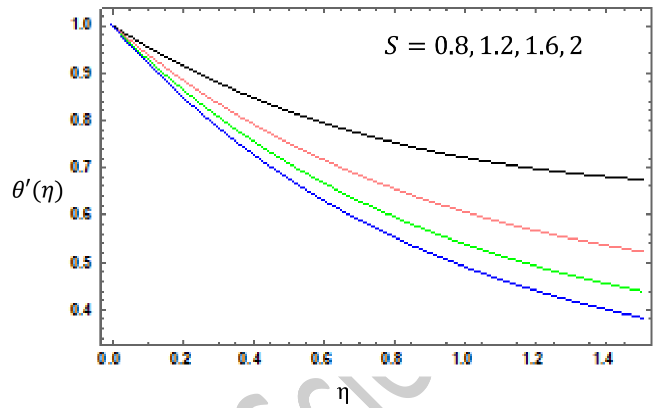

Figure 10 : Variation of unsteadiness parameter S on the temperature profile of Casson fluid

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