

# AN IMPROVED PARTICLE SWARM OPTIMIZATION TECHNIQUE FOR SELECTIVE HARMONIC ELIMINATION IN CASCADED-H BRIDGE INVERTER

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**Abstract:** Multilevel Inverters (MLI) are put-to-use in a wide range of applications viz. DC power source utilization, Electric Vehicle drives, and Power factor compensators. Generation of high-quality output voltage waveforms stands as a vital requisite of MLI in such applications. In this paper, Cascaded H-Bridge (CHB) multilevel inverter topology has been implemented in which fifth, seventh harmonics in a seven-level CHB are being eliminated using the Selective Harmonic elimination technique. SHE technique involves solving non-linear transcendental equations which gets complicated if conventional algebraic methods and iterative methods are employed. To simplify these calculations Particle Swarm Optimization (PSO) method has been employed in this paper. PSO, Constriction factor-based PSO (CF-PSO), Multi-swarm Multi-velocity (MMPSO) have been used to generate switching angles for five-level CHB inverter and seven-level CHB inverter. The switching angles generated using MMPSO have been used in the simulation of seven-level CHB in MATLAB/Simulink software environment. The analysis shows that the optimal solution has been tracked at a lesser number of iterations using MMPSO when compared to PSO and CF-PSO.

**Keywords—** Cascaded H-bridge Inverter, Constriction factor-based PSO, Multilevel Inverter, Multi-swarm Multi-velocity PSO, Particle Swarm Optimization, Selective Harmonic Elimination.

## I. INTRODUCTION

A Power electronic device which converts DC power to AC power is known as Inverter. Conventional Inverter produces two voltage levels, one positive level and other is negative level whereas multilevel inverter generates more than two levels. The conventional two-level multilevel inverter operates at higher switching frequencies which cause high switching losses. They also have high harmonic distortions and high (dv/dt) stress on the switches. These drawbacks initiated for the development of new topology. Concept of generating multiple voltage levels has come to light around 1975 and a gained lot of importance in high power and medium voltage applications. Multilevel inverters produce output with a low total harmonic distortion and switches face a very low dv/dt stress when compared to the conventional topology. [1], [2].

Advantages of MLI are as follows

1. Increased number of voltage level.
2. Lesser harmonic distortions.
3. Reduced switching losses.
4. Operates at a higher frequency and also the fundamental frequency.
5. Generation of a higher level of voltage with reduced device rating.
6. Improves the quality of staircase waveform.[2]

To control the output of multilevel inverters and also to eliminate harmonics and minimize THD, there are several modulation techniques such as PWM, SPWM, SVPWM, SHE –PWM.

Using PWM, SVPWM one can mitigate the harmonics whereas, using SHE-PWM lower order harmonics can be eliminated. SHE-PWM has gained a lot of importance in high power, high voltage applications where reducing switching losses is of key importance. SHE-PWM technique is dependent on the decomposition of voltage/current waveform using Fourier series. [5]

In SHE-PWM technique the harmonic content generated can be calculated before and can be optimized as much as possible. However, the complexity in implementing SHE technique is solving the nonlinear transcendental equations, which are the result of Fourier analysis of output waveforms. Several solving techniques as, numerical methods, optimisations techniques, resultant theory, algebraic methods have been implemented to attain switching angles for different types of waveforms. [5]

Numerical methods are simple in implementation but require a proper initial estimation for fast convergence. Even with a good initial guess, this method is restricted to a system where there are few switching angles to be calculated. As the number of switching angles increase the difficulty in convergence increases. [3]

The algebraic methods like Groebner bases, symmetric polynomial, equal area-based technique, first convert transcendental expressions to polynomial expressions and then solve these polynomials to obtain switching angles. As the number of switching angles increases polynomial order increases and makes it cumbersome to solve the equations. [5]

The optimisation based method such as Particle swarm optimisation, genetic algorithms, artificial neural networks, ant colony systems minimizes the cost function to obtain switching angles. These methods depend on the search area and assumed parameters in the algorithm. [3], [5].

PSO is an optimization-based technique to solve the SHE-PWM equations. When compared to other optimisation techniques PSO has been proved to provide efficient performance at higher inverter levels also.[6] The drawback of PSO is premature convergence that is falling into local optimum. Studies show that enhancing the diversity of the population should be the remedy for this problem. Multi swarm Multi-velocity PSO consists of the velocity update methods and three-position update methods. This increases the diversity of the particles and also generates optimum results at a lesser number of iterations when compared to traditional PSO [9], [10].

The paper is arranged as follows, section one briefs about Cascaded H-bridge inverter and its operation. A detailed explanation of SHE problem is given in section 1. Section two deals with the PSO, CF-PSO, MMPSO working algorithm and parameters used. Simulation results and comparisons are discussed in section three and conclusions of the paper in section four.

**CASCADED H-BRIDGE MULTILEVEL INVERTER**

Cascaded h bridge is the popular topology of all the available topologies of multilevel inverter because of its simplicity. More than one H-Bridge is connected in series in cascaded h bridge inverter. Each bridge is having an isolated DC source as input. Below circuit diagram is the general circuit diagram for n level inverter.

Single H-bridge produces three different levels of output voltage  $V_{dc}$ ,  $-V_{dc}$  and zero levels when the switches are triggered in a particular fashion. The final output is a staircase waveform which is the sum of the individual output of each H-Bridge.

$$V_o = V_1 + V_2 + V_3 \dots V_s$$

s is the number of DC sources being used.

$$m = 2s + 1$$

Using s number of dc sources m level voltage can be synthesized. For instance, a seven-level CHB we need three DC sources that is, Three H Bridges connected in series.

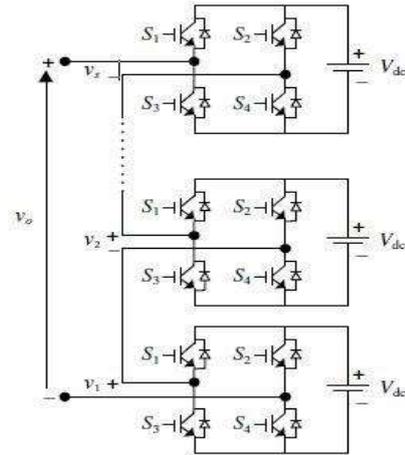


Fig.1 Circuit diagram of Cascaded H-Bridge Inverter.

**SELECTIVE HARMONIC ELIMINATION**

The number of switching angles is equal to the number of DC sources being used in multilevel inverters. S-1 number of harmonics can be eliminated at maximum, where s is the number of DC sources.

Fourier series expansion for staircase output in fig 2 is

$$V_o = \sum_{n=1,3,5,\dots} V_n \sin(n\omega t) \tag{1}$$

Where  $\omega = 2\pi f$  and f is fundamental frequency and  $V_n$  is defined as below.

$$V_n = \frac{4V_{dc}}{n\pi} \sum_{i=1,2,3,\dots}^s \cos(n\theta_i) \tag{2}$$

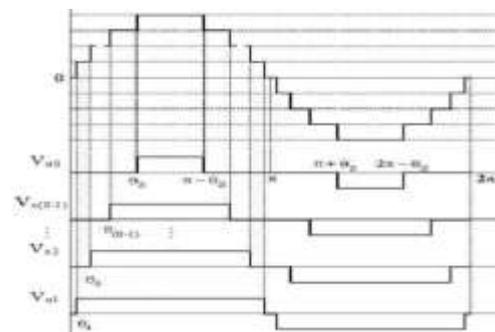


Fig. 2 Staircase output of Cascaded H-Bridge Inverter.

Harmonic elimination is done by calculating switching angles  $\theta_1, \theta_2, \theta_3 \dots \dots \dots \theta_s$ , which makes output voltage equal to fundamental voltage and suppress other harmonic voltages of order .switching angles should satisfy the below condition

$$0 \leq \theta_1 \leq \theta_2 \leq \theta_3 \dots \dots \dots \leq \theta_s \leq \frac{\pi}{2} \dots \dots \dots \tag{3}$$

For example, in a seven-level Cascaded H-bridge inverter there are three DC sources and we can eliminate fifth and seventh harmonics by calculating from the following set of equations. These mathematical equations are formed from equations (1) and (2).

$$\cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3) = sm_a$$

$$\begin{aligned} \cos(5\theta_1) + \cos(5\theta_2) + \cos(5\theta_3) &= 0 \\ \cos(7\theta_1) + \cos(7\theta_2) + \cos(7\theta_3) &= 0 \dots\dots (4) \end{aligned}$$

Where  $m_a$  is modulation index and  $s$  is the number of dc voltage sources.

$$m_a = \frac{V_1}{V_{1max}}$$

Where  $V_1$  is the desired fundamental voltage and  $V_{1max}$  is as defined below.

$$V_{1max} = \frac{4sV_{dc}}{\pi}$$

$V_{dc}$  is the magnitude of the DC voltage source. The above set of equations can be solved by using PSO, CF-PSO, MMPSO and determine required angles.

II. METHODOLOGY

PARTICLE SWARM OPTIMIZATION

Particle swarm optimization was first proposed in 1995 by Kennedy and Eberhart. Social behaviour of animals like bird flocking was the inspiration for this concept. Each particle in the population gets randomly distributed in the D-dimensional search space. Each particle has its position and velocity. Particles are driven towards the optimal solution based on their history and also on the information from the neighbour particles. Each particle is described by two vectors, position and velocity vectors.

$$\begin{aligned} X_i &= [x_{i1}, x_{i2}, x_{i3} \dots x_{iD}] \\ V_i &= [v_{i1}, v_{i2}, v_{i3} \dots v_{iD}] \end{aligned}$$

$i=1, 2, 3, \dots, n$ . Where  $n$  is the size of the population. Each particle's position and velocity gets updated based on their personal best and also the global best which best of the entire population as shown below.

$$v_{ij}(It + 1) = wv_{ij}(It) + c_1r_1(x_{ij}^p(It) - x_{ij}(It)) + c_2r_2(x_{ij}^g(It) - x_{ij}(It)) \quad (5)$$

$$x_{ij}(It + 1) = x_{ij}(It) + v_{ij}(It + 1) \quad (6)$$

Where,  $j = 1, 2, 3, \dots, D$ .  $c_1$  and  $c_2$  are the cognitive acceleration factor and social acceleration factor,  $w$  is the inertia weight.  $r_1$  and  $r_2$  are random values in range [0, 1].  $x_{ij}^p$  is the best position of the particle and  $x_{ij}^g$  is the best position in the entire population.

To obtain optimal switching angles using PSO, the following steps can be implemented

1. The algorithm begins with the setting of population size ( $n$ ) and initializing them with a random position within 0 and  $\pi/2$  and velocity in limits  $V_{max}$  and  $V_{min}$ . Dimension  $D$  of the search

space is the same as the number of switching angles per quarter cycle. Set the maximum number of iterations (MaxIt).

2. Calculate the fitness value of every individual particle. Our objective is to maximize the fundamental voltage and minimize the specific harmonics as  $V_5, V_7$ . Fitness function is formulated as below

$$ff = \min \left\{ \left( 100 \frac{V_1^* - V_1}{V_1^*} \right)^4 + \sum_{n=5,7,\dots}^{\sigma} \frac{1}{n} \left( 50 \frac{V_n}{V_1} \right)^2 \right\} \quad (7)$$

$\sigma = 3s - 2$  If  $S$  is odd and  $\sigma = 3s + 1$  if  $s$  even.

Harmonic order that can be eliminated is of order  $6k \pm 1$ .

3. If the  $i^{th}$  particle position is better than the  $x_{ij}^p$  then update  $x_{ij}^p$  with current position .similarly update  $x_{ij}^g$  with the current best position among all the personal bests of the population.
4. Position and velocity vectors are updated using equations (5), (6).
5. Repeat steps 2, 3, and 4 if the current iteration is less than the maximum iteration.

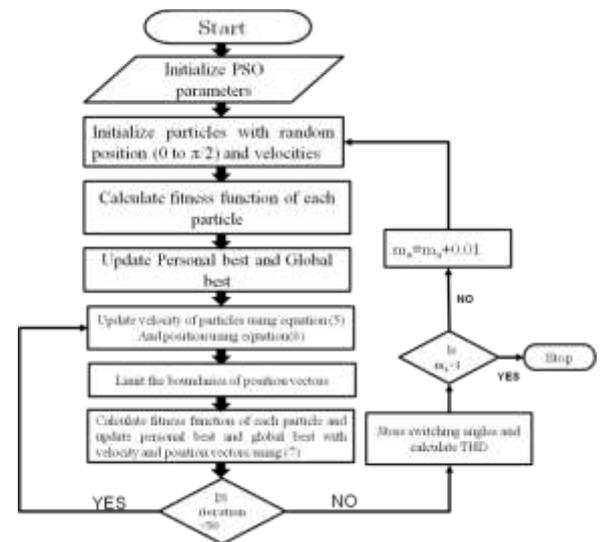


Fig.3 Flowchart of PSO

CONSTRICTION FACTOR BASED PSO

The traditional PSO is slow at convergence while solving a complex objective problem. One of the modifications to overcome this is including constriction factor in velocity update equation. Constriction factor improves the convergence characteristics.

$$v_{ij}(It + 1) = \kappa \{ wv_{ij}(It) + c_1r_1(x_{ij}^p(It) - x_{ij}(It)) + c_2r_2(x_{ij}^g(It) - x_{ij}(It)) \} \quad (8)$$

$\kappa$  is defined as below

$$\kappa = \frac{2}{|2 - c - \sqrt{c^2 - 4c}|} ; c = c_1 + c_2 > 4; \quad (9)$$

### MULTI-SWARM MULTI VELOCITY PARTICLE SWARM OPTIMIZATION

The traditional PSO consists of single swarm and has a single velocity update method.  $c_1 r_1 (x_{ij}^p (It) - x_{ij} (It))$ ,  $c_2 r_2 (x_{ij}^g (It) - x_{ij} (It))$  are the two terms which decide the direction of the particles in traditional PSO. When particles come close to the optimal position the moving distance of particles gradually decreases and this results in getting trapped into local optimum. Other demerits of PSO are low precision and reliability when applied to high dimensional search area problems. To address all these demerits, an improved PSO algorithm, MMPSO is proposed in this paper which consists of multiple particle swarms and multiple velocity update methods. Multiple particles swarms exchange information among them and by using multiple velocity update methods diversity of the particles increase and save algorithm from falling into local optimum. MMPSO can greatly improve the precision of the algorithm.

There are three particle swarms and three velocity update methods in MMPSO. Main particle swarm, Global auxiliary particle swarm, Local auxiliary group are the three groups in MMPSO. Velocity update method with improved inertia weight, Constriction factor-based velocity update method and random velocity update are the three kinds of velocity update methods being implemented in MMPSO.

#### a) Main particle swarm

The velocity update method for the main particle swarm is with improved inertia weight as shown below

$$v_{ij} (It + 1) = wv_{ij} (It) + c_1 r_1 (x_{ij}^p (It) - x_{ij} (It)) + c_2 r_2 (x_{ij}^g (It) - x_{ij} (It)) \quad (10)$$

$$x_{ij} (It + 1) = x_{ij} (It) + v_{ij} (It + 1) \quad (11)$$

$x_{ij} (It)$ ,  $v_{ij} (It)$  are the position and velocity of the  $i^{\text{th}}$  particle,  $x_{ij}^p (It)$  is personal best of the particle,  $x_{ij}^g (It)$  is the global best of all particles.  $c_1, c_2$  are cognitive and social factors.  $r_1, r_2$  are random values between [0, 1].  $w$  is the improved inertia weight given as

$$w = w_e \left( \frac{w_e}{w_s} \right)^h + \frac{w_e (0.5 - r_3)}{k w_s} \quad (12)$$

$$h = \frac{1}{1 + k \left( \frac{It}{MaxIt} \right)} \quad (13)$$

$w_e, w_s, k$  are constant-coefficient  $w_e, w_s$  are minimum and maximum inertia weight coefficients,  $MaxIt$  is the maximum number of iteration.  $r_3$  is the random values in range[0, 1].

#### b) Global Auxiliary Particle Swarm

The velocity of the particles in this group is updated using Constriction factor-based velocity update method and also random velocity update method. Only one method is chosen to update the velocity of particles at a time with a certain probability.

##### 1) Constriction Factor velocity update method:

Velocity update equation is as below

$$v_{2ij} (It + 1) = \kappa \left\{ wv_{2ij} (It) + c_4 r_4 (x_{2ij}^p (It) - x_{2ij} (It)) + c_5 r_5 (x_{2ij}^g (It) - x_{2ij} (It)) + \frac{(0.5 - r_6) (x_{2ij}^g (It) - x_{2ij}^p (It))}{1 + e^{-ffi}} \right\} \quad (14)$$

$v_{2ij} (It)$ ,  $x_{2ij} (It)$  are velocity and position of particle.  $x_{2ij}^p (It)$ ,  $x_{2ij}^g (It)$  are the personal best and global best of particles.  $r_4, r_5, r_6$  are random values in [0, 1].  $ffi$  is the fitness value of the current iteration.  $\kappa$  is constriction factor.

$$\kappa = \frac{2}{|2 - c - \sqrt{c^2 - 4c}|} ; c = c_4 + c_5 > 4;$$

##### 2) Random velocity update method:

Velocity equation for this method is as follows

$$v_{2ij} (It + 1) = v_{2ij} (It) + Fx (It) - G (It) \quad (15)$$

$$F = [a_1 \ a_2 \ a_3 \dots a_j]^T$$

$$x (It) = [x_{2i1} \ x_{2i2} \ x_{2i3} \dots x_{2ij}] \quad (16)$$

$$G (It) = [a_1 V_{max} \eta_1 (It) \ a_2 V_{max} \eta_2 (It) \dots a_j V_{max} \eta_j (It)]^T$$

Where

$$a_j = 0.5 - r_j \quad (17)$$

$$\eta_j (It) = \frac{x_{2ij} (It)}{x_{max} - x_{min}} \quad (18)$$

Position update equation for both methods is

$$x_{2ij} (It + 1) = x_{2ij} (It) + v_{2ij} (It + 1) \quad (19)$$

c) Local auxiliary particle swarm

There is only the position update method for this group of particles.

$$x_{3ij}(It) = x_{ij}^g(It) + (0.5 - r)x_{ij}^g(It) \quad (20)$$

MMPSO algorithm can be implemented as follows

- Main particle swarm formation: other two particle swarms are constructed from the main group. As in traditional PSO set population and initialize particles with random velocities and position within their boundaries. Evaluate the fitness value of particles with help of equation (7).
- Global Auxiliary Particle swarm formation: Arrange the particles of the main group in descending order of their fitness values. Then the top fifty per cent of the particles with large fitness values form a new group. These new group particles are replicated and added to the new group and collectively called Global auxiliary swarm. Hence there are equal numbers of particles in main and global auxiliary groups.

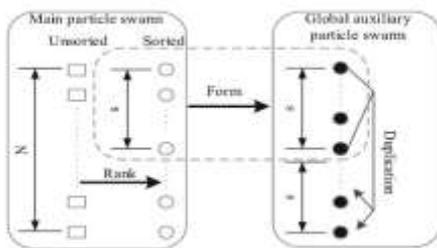


Fig. 4 Global Auxiliary group formation

N are unsorted particles, S are sorted particles.

- Velocity and position of the main group are always updated by using equation (10) and (11).
- Velocity and position of Global auxiliary group are always updated by equations (14) and (20) or (15) and (20).
- When the iteration number is greater than ten, the particles of the main group with large fitness value, undergo mutation mechanism using below given equation

$$x_{new}(It) = (1.5 - 3r)x_w(It) \quad (21)$$

- For every iteration whose number is in between  $0.4 \cdot \text{MaxIt}$  and  $0.9 \cdot \text{MaxIt}$ , excellent particles in global auxiliary replace worst particles of the main group.
- Local Auxiliary Particle Swarm construction: Once the iteration of the optimization algorithm is greater than the  $0.9 \cdot \text{MaxIt}$ , the local auxiliary group is constructed. The personal best of all particles from the main group constructs local auxiliary group. This group searches near the best of all the particles and provides better information

to the main swarm. Best particles of the local auxiliary group are swapped with current worst particles of the main group for each iteration after  $0.9 \cdot \text{MaxIt}$  till  $\text{MaxIt}$  and end.

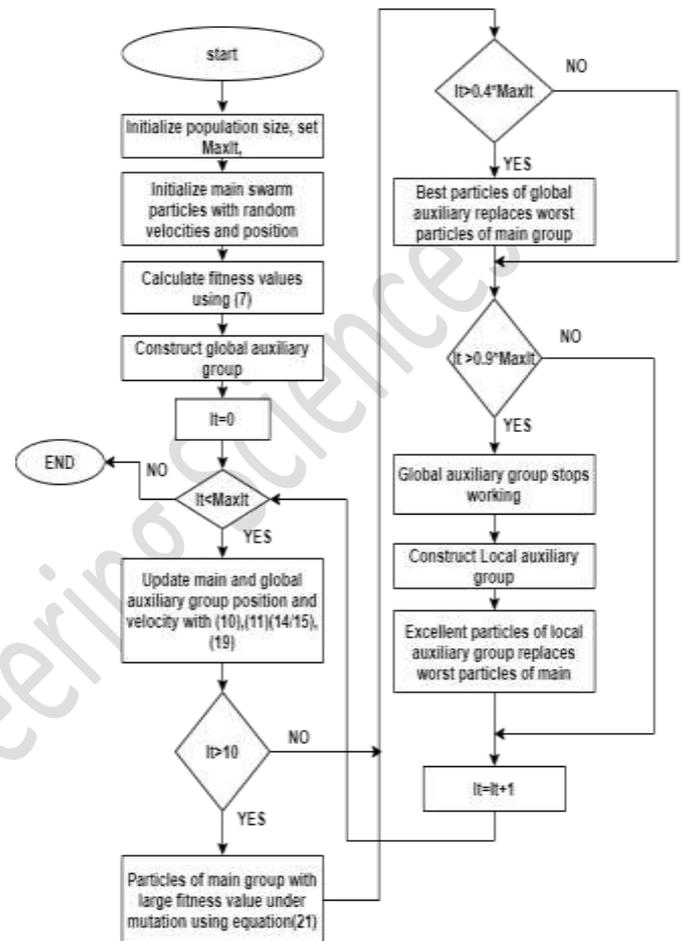


Fig. 5 MMPSO flowchart.

The same algorithm is implemented for Modulation index from 0.2 to 1 with a step increment of 0.01. For each modulation index, store corresponding switching angles. The three algorithms are coded in MATLAB R2014a environment. The parameter of all algorithms are listed in TABLE I

TABLE I  
Parameters of PSO, CF-PSO, MMPSO

Algorithm	Parameters
PSO	Population size:50 MaxIt=50; V <sub>dc</sub> =100 volts W=0.9; C <sub>1</sub> =C <sub>2</sub> =2
CF-PSO	Population size:50 MaxIt=50 V <sub>dc</sub> =100 volts C <sub>1</sub> =C <sub>2</sub> =2.01
MMPSO	Population size:50 MaxIt =50 C <sub>1</sub> =C <sub>2</sub> =2

	$C_4=C_5=2.05$ $W_s = 1.65$ $W_e = 0.53$ $k = 100$
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III. RESULTS

SIMULATION RESULTS

To verify the correctness of the MMPSO algorithm, a seven-level H bridge inverter as shown in fig1 has been modelled in the MATLAB/Simulink environment. The value of the three DC sources is equal to 100V. The angles used in this simulation are calculated with the help of MMPSO. At modulation index 0.8 angles are angle1=11.5042, angle2= 28.7172, angle3=57.1062. The staircase output voltage waveform with these switching angles is shown in fig. (7). Gate pulses for the switches S<sub>1</sub> of each bridge are shown in Figure (6).

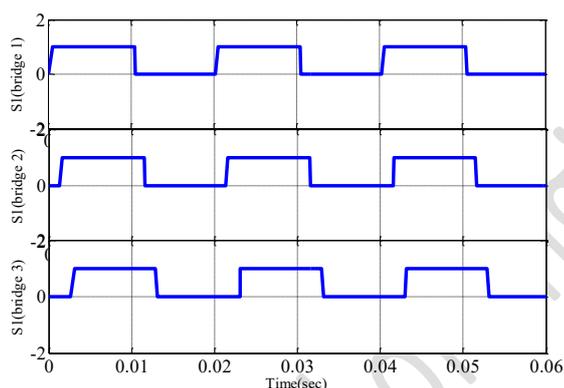


Fig. 6 Gating pulses.

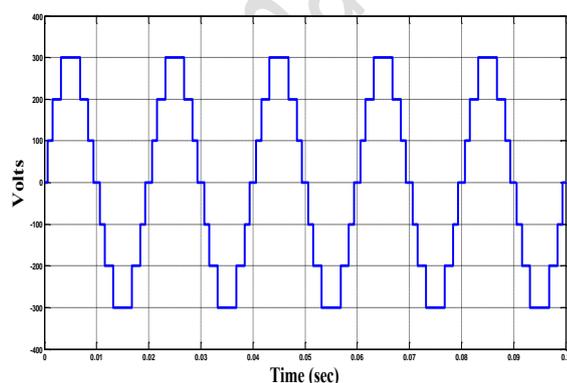


Fig. 7 Output voltage of Seven-level inverter.

The FFT analysis for the above staircase output gives THD% as 9.60% and fifth, seventh harmonic percentages are 0.00% and 0.00% with fundamental voltage as 305.6

peak(216 Rms). Figure (10b) clearly shows that selected harmonics are eliminated.

Optimal switching angles for modulation index ranging from 0.2 to 1 are plotted in figure (9).

The switching angles obtained from PSO, CF-PSO, MMPSO are also used in the simulation at the same modulation index 0.8. The FFT analysis shows that THD% is almost the same but selected harmonic percentages are not eliminated. Using PSO THD% is 11.11%, a fifth harmonic percentage if 0.11%; the seventh harmonic percentage is 2.52%, which shows that MMPSO obtained optimal switching angles when compared to PSO. TABLE II shows the THD% and harmonic percentages when PSO, CF-PSO, MMPSO are used to calculate switching angles at 0.8 modulation index. Various other factors are also listed in Table II

TABLE II  
Comparisons of PSO, CF-PSO, MMPSO

Algorithm	PSO	CF-PSO	MMPSO
Modulation Index	0.8	0.8	0.8
THD	11.11%	9.34%	9.60%
H5%	0.11%	0.37%	0.00%
H7%	2.52%	1.72%	0.00%
Time for 50 iterations(sec)	0.411220	0.358662	1.958872
Code complexity	Low	Low	High
Fitness value	0.2321	0.2202	4.8351e-12

With CF-PSO, THD% has decreased, but selected harmonics are still present. The fitness value using MMPSO has been highly minimized when compared to the other two algorithms. With MMPSO, the minimal fitness value has been achieved with a lesser number of iterations. Run time for MMPSO algorithm has increased because of three different velocity update methods. Run time of CF-PSO is less than PSO which shows CF-PSO improved the convergence characteristics. Simulation results show that MMPSO increased the exploration area of the algorithm and eliminated the fifth and seventh harmonics.

In this paper, a Seven-level Cascaded H-bridge Inverter has been simulated in MATLAB/Simulink R2014a environment. Selective harmonic elimination technique has been implemented to obtain the optimal switching angles which can eliminate fifth and seventh harmonics from the output voltage. PSO, CF-PSO, MMPSO have been used to solve the SHE problem. Results illustrate CF-PSO has faster convergence and MMPSO has explored the search area more efficiently when compared to PSO and CF-PSO. Accuracy and precision have been improved with MMPSO.

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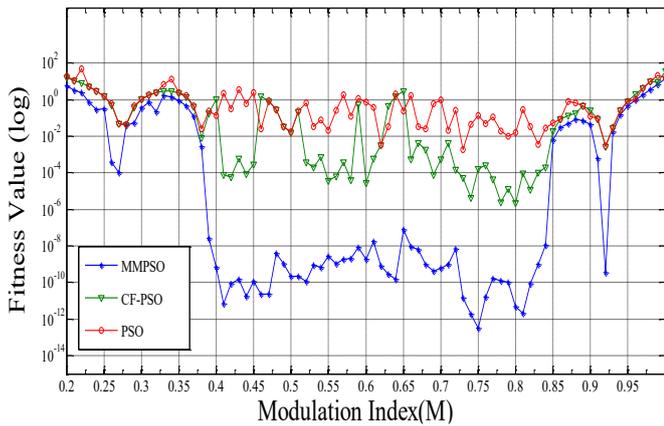


Fig.8.Fitness values Versus Modulation Index.

From figure (8), it is evident that MMPSO can minimize fitness value range to  $[10^{-12}, 10^{-8}]$  for about 70% of the modulation index range.

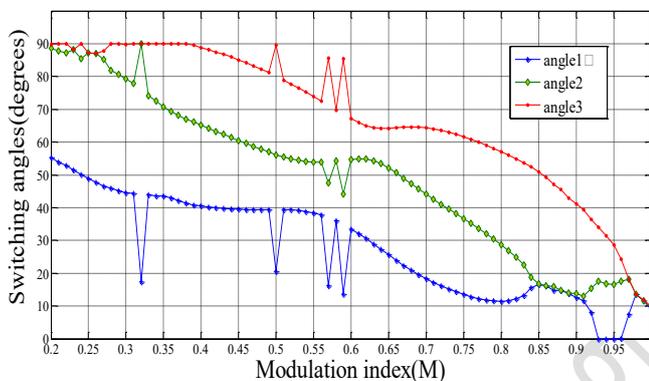
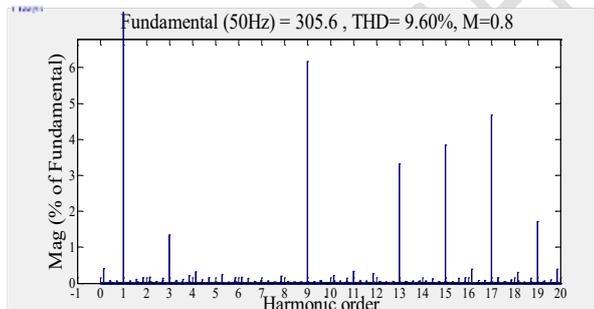
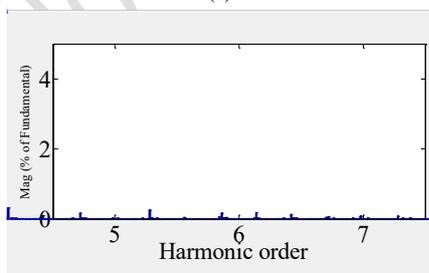


Fig.9.Switching angles versus Modulation Index.



(a)



(b)

Fig.10 (a) FFT analysis of staircase output of Seven-level inverter. (b)The zoomed spectrum of FFT analysis.

IV. CONCLUSIONS