

STUDY OF A TRANSPORTATION PROBLEM OF AN ESSENTIAL ITEM FROM VARIOUS ORIGINS TO DIFFERENT DESTINATIONS BY USING EQM

Dr. M. RAJESWARA REDDY,

*HOD, Associate Professor, H & S Department, Samskruti College of Engineering and Technology, Ghatkesar, Hyderabad.
E-mail: mrajesh3424@gmail.com*

ABSTRACT

We considered a transportation problem of essential item rice from the different origins to different destinations and formulated the problem as a LPP model. We obtained an IBFS to the problem by NORTH-WEST CORNER METHOD (NWCM) and EASY QUICK METHOD (EQM) and compared the results and displayed in the tables. The key idea in EQM is to minimize the best combinations of the solution to reach the optimal solution. Comparatively, applying the EQM in the proposed method obtain the best initial feasible solution to the transportation problem and performs faster than the existing methods with a minimal computation time and less complexity. The proposed method is therefore an attractive alternative to traditional problem solution methods. The EQM can be used successfully to solve all different transportation problems in different areas of study. Finally, MODI method has applied for optimal solution for the effectiveness of the proposed method.

Keywords— Transportation Models, Initial Basic Feasible Solution (IBFS), Optimal Solution, EASY QUICK METHOD (EQM) and NORTH-WEST CORNER METHOD (NWCM).

I. INTRODUCTION

Industries require planning in transporting their products from production centers to the users end with minimal transporting cost to maximize profit. This process is known as Transportation Problem which is used to analyze and minimize transportation cost. This problem is well discussed in operation research for its wide application in various fields, such as scheduling, personnel assignment, product mix problems and many others, so that this problem is really not confined to transportation or distribution only. In the solution procedure of a transportation problem, finding an Initial Basic Feasible Solution (IBFS) is the prerequisite to obtain the optimal solution. Again, development is a continuous and endless process to find the best among the bests. The growing complexity of management calls for development of sound methods and techniques for solution of the problems. Considering these factors, this research aims to propose an algorithm “EQM” to obtain an IBFS for the transportation problems. Several numbers of numerical problems are also solved to justify the method. Obtained results show that the proposed algorithm is effective in solving transportation problems.

The Transportation problem is one of the traditional functions of the Linear Programming Problems. Transportation model provides a greater impact on the transportation of the commodities from the manufacturing places. The basic Transportation problem was initially proposed by Hitchcock. Transportation problem is being prioritized in both service and manufacturing industries. It is a network optimization problem which is well known in operation research. This problem is not only used in transportation problem but also in various fields like scheduling, personnel assignment, product mix problems, etc. Now a day's transportation problem has become a standard application for industrial organizations having several manufacturing units, warehouses and distribution centers. Following figure: 1 shows the transportation network.

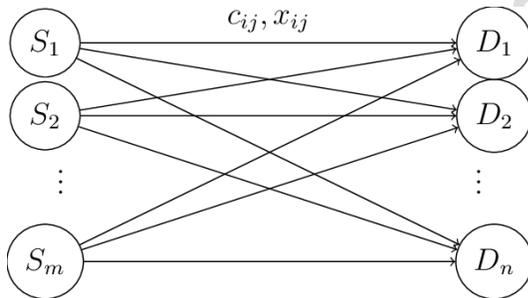


Figure: 1 - Network of T.P.

A Transportation problem is one of the earliest and most important applications of LPP. Description of a classical transportation problem can be given as follows. A certain amount of homogeneous commodity is available at number of sources/origins and a fixed amount is required to meet the demand at each number of destinations/distribution centers. Then finding an optimal schedule of shipment of the commodity with the satisfaction of

demands at each destination is the main goal of the problem. In 1941, Hitchcock [1] developed the basic transportation problem along with the constructive method of solution and later in 1949 Koopmans [2] discussed the problem in detail. Again in 1951 Dantzig [3] formulated the transportation problem as LPP and also provided the solution method. The transportation problem in which the objective is to minimize the total cost of shipping a single commodity from a number of sources (m) to a number of destinations or sinks (n). Because of the special structure of the transportation problem, a special algorithm can be designed to find an optimal solution efficiently. The most important and successful applications in the optimization refers to transportation problem (TP).

The basic steps for obtaining an optimum solution to a transportation problem are: (P. K. Gupta and Man Mohan, (2003) (11).

- Step 1: Mathematical model of the problem
 - Step 2: Finding an Initial Basic Feasible Solution (IBFS)
 - Step 3: To test whether the solution is an optimal one or not.
- If not, improve it further till the optimality is achieved.

Most of the time the initial basic feasible solution of transportation problem is calculated by using the methods of North-West Corner Method or Least-Cost Method or Vogel's Approximation Method, and then finally the optimality is checked by MODI (Modified Distribution Method). Many researchers have provided improved or different algorithm to solve transportation problem. In last few years Hakim, M. A. (2012) [9], Khan, A. R. (2011) [6, 14], Azad, S. M. A. K. et al. (2017) [7], Ahmed, M. M., et. al (2016) [8], Raigar, S. and

Modi, D. G. (2017) [9], Hosseini, E. (2017) [10] developed new methods to determine Initial Basic Feasible Solution of transportation problem.

In this paper, we considered a transportation problem of an essential item, rice, from the different origins to different destinations and formulated the problem as a LPP model. We obtained an IBFS to the problem by NWCM and EQM and compared the results and displayed in the tables. The key idea in EQM is to minimize the combinations of the solution by choosing the best least cells to reach the optimal solution. Comparatively, applying the EQM in the proposed method obtains the best Initial Basic Feasible Solution to a transportation problem and performs faster than the existing methods with a minimal computation time and less complexity. The proposed method is therefore an attractive alternative to traditional problem solution methods. The EQM can be used successfully to solve all different transportation problems in different areas of study. Finally, MODI method has applied for optimal solution for the effectiveness of the proposed method.

II. MATHEMATICAL MODEL OF THE PROBLEM

In this problem goods are transported from a set of origins to a set of destinations subject to the supply and demand of the origin and destination respectively, such that the total cost of transportation is minimized.

Consider a homogeneous commodity, rice, is transported from the different origins (O_1 , O_2 , and O_3) to the different destinations (D_1 , D_2 , D_3 and D_4). Let a_i represents the

amount of the rice is available at i^{th} origin with $a_1 = 75$ quintals, $a_2 = 60$ quintals, $a_3 = 40$ quintals. Similarly, Let b_j represents the amount of the rice is required at j^{th} destination with $b_1 = 30$ quintals, $b_2 = 65$ quintals, $b_3 = 55$ quintals and $b_4 = 25$ quintals. The transportation problem has shown below table 1.

Table 1: Transportation problem

	D_1	D_2	D_3	D_4	Supply(a_i)
O_1	11	22	6	5	75
O_2	16	31	14	15	60
O_3	5	21	4	9	40
Demand(b_j)	30	65	55	25	

Let C_{ij} be the unit transportation cost (in Rs.) per a quintal of rice from O_i to D_j , the requirements of the destinations D_j , $j = 1, 2, 3, 4$ must be satisfied by the supply of available units at the points of origin O_i , $i = 1, 2, 3$ if X_{ij} is the number of units that are shipped from O_i to D_j , then the problem in determining the values of the variables X_{ij} , $i = 1, 2, 3$ and $j = 1, 2, 3, 4$ should minimize the total of the transportation/shipping costs.

The main objective of transportation problem is to determine the amount of the commodity X_{ij} transported from the origins to the destinations so as to minimize the total transportation cost.

The total transportation cost (Z) in the above problem is

$$\text{Min. } Z = 11X_{11} + 22X_{12} + 6X_{13} + 5X_{14} + 16X_{21} + 31X_{22} + 14X_{23} + 15X_{24} + 5X_{31} + 21X_{32} + 4X_{33} + 9X_{34} .$$

Subject to the constraints,

$$X_{11} + X_{12} + X_{13} + X_{14} = 75$$

$$X_{21} + X_{22} + X_{23} + X_{24} = 60$$

$$X_{31} + X_{32} + X_{33} + X_{34} = 40$$

$$X_{11} + X_{21} + X_{31} = 30$$

$$X_{12} + X_{22} + X_{32} = 65$$

$$X_{13} + X_{23} + X_{33} = 55$$

$$X_{14} + X_{24} + X_{34} = 25$$

$$X_{ij} \geq 0, i = 1, 2, 3 \text{ and } j = 1, 2, 3, 4$$

Mathematically, the transportation problem can be represented as a linear programming model. Since the objective function and the constraints are linear functions and that can adopt the linear programming (LP) technique with equality constraints.

LP technique can be used in different product areas such as oil plum industry [4, 13]. However, The LP technique can be generally used by genetic algorithm such as Sudha at el. article [5, 12]. The transportation solution problem can be found with a good success in the improving the service quality of the public transport systems [6, 11]. As well as, the transportation solution problem is used in the electronic commerce where the area of globalization the degree of competition in the market article [7, 10], and many other fields.

However, there are several different algorithms to solve transportation problem that represented as LP model. Among these are the known algebraic procedures of the simplex method, which may not be the best method to solve the problem. Therefore, more efficient and simpler procedures have been improved to solve transportation problems. Typically, the standard scenario for solving transportation problems is working by sending units of a product across a network of highways that connect a given set of cities. Each city is considered as a source (S) in that units will be shipped out from, while units are demanded there when

the city is considered as a sink (D). In this scenario, each sink has a given demand, the source has a given supply, and the airway that connects source with sink as a pair has a given transportation cost/(shipment unit).

The problem is to determine an optimal transportation scheme that is to minimize the total of the shipments cost between the nodes in the network model, subject to supply and demand constraints. As well as, this structure arises in many applications such as; the sources represent warehouses and the sinks represent retail outlets. Moreover, Ad-hoc networks are designed dynamically by group of mobile devices. In Ad-hoc network, nodes between source and destination act as a routers so that source node can communicate with the destination node [8,9].

III. IBFS USING NORTH-WEST CORNER METHOD (NWCN)

There are several methods for finding an initial basic feasible solution of the transportation problems which are based on different of special linear programming methods, among these are: North-West Corner method (NWCN), Least cost method (LCM), Vogel's approximation method (VAM), Row Minimum Method (RMM), Column Minimum Method (CMM) etc.,. Basically, these methods are different in term of the quality for the produced basic starting solution and the best starting solution that yields minimize the objective function value. North-West Corner method (NWCN) is one of the conventional methods that give better Initial Basic Feasible Solution (IBFS) of a Transportation Problem (TP). This method is very effective as it provides step by step solution and it is very simple to find IBFS through this method. Following table shows the IBFS by using NWCN.

Table 2: IBFS by using NWCM.

	D1	D2	D3	D4	Supply(ai)
O1	30	45			75
	11	22	6	5	
O2		20	40		60
	16	31	14	15	
O3			15	25	40
	5	21	4	9	
Demand(bj)	30	65	55	25	

Total transportation cost by using NWCM is, $30 \times 11 + 45 \times 22 + 20 \times 31 + 40 \times 14 + 15 \times 4 + 25 \times 9 = \text{Rs.}2785$

IV. IBFS USING EASY QUICK METHOD (EQM)

Step1: We must check the matrix balanced or not, if the total supply is equal to the total demand, then the matrix is balanced and also apply Step 2. If the total supply is not equal to the total demand, then we add a dummy row or column as needed to make supply is equal to the demand. So the transportation costs in this row or column will be assigned to zero.

Table 3: Balanced Transportation problem

	D1	D2	D3	D4	Supply(ai)
O1	11	22	6	5	75
O2	16	31	14	15	60
O3	5	21	4	9	40
Demand(bj)	30	65	55	25	175

Step2: Selecting the best least cost two cells in each row and marks them by circle as shown in below Table 4.

Table 4: Selection of least cost cells in rows

	D1	D2	D3	D4	Supply(ai)
O1	11	22	⑥	⑤	75
O2	16	31	⑭	⑮	60
O3	⑤	21	④	9	40
Demand(bj)	30	65	55	25	175

Step3: Selecting the best least cost two cells in each column (for not having two circles marked) and marks them by circle as shown below in Table 5

Table 5: Selection of least cost cells in columns.

	D1	D2	D3	D4	Supply(ai)
O1	⑰	22	⑥	⑤	75
O2	16	31	⑭	⑮	60
O3	⑤	21	④	9	40
Demand(bj)	30	65	55	25	175

Step4: Now identifying each row and allocate commodity as much as possible with least unit cost in the marked cells. Also adjust the supply and demand and cross out the satisfied row or column. If both row and column satisfied then cross out both corresponding row column. Repeat the process until the all the commodity should be allocated as per requirement and demand.

Table 6: Final allocation of the commodities in the cells

	D1	D2	D3	D4	Supply(ai)
O1	⑪	50 22	⑥	25 ⑤	75
O2	16	5 31	55 ⑭	⑮	60
O3	30 ⑤	10 21	④	9	40
Demand(bj)	30	65	55	25	175

The total transportation cost by using EQM is, $50*22 + 25*5 + 5*31 + 55*14 + 30*5 + 10*21 = \text{Rs. } 2510$ which is better IBFS comparatively with the solution obtained by NWCM.

V. MOVING TOWARDS OPTIMALITY

In order to verify whether the above IBFS using NQM is optimal solution or not, we can apply MODI (Modified Distribution method) or u-v method as follows:

Step-1: Determine an initial basic feasible solution as we derived in the above problem by New Quick Method. Then take the costs only of the occupied cells which are basic cells and corresponding variables are basic variables. If the number of basic cells is equal to $m+n-1$, then IBFS is non-degenerate. Otherwise degenerate, make degenerate as non-degenerate by assigning small number ϵ in the required number of unoccupied cells.

Step-2: Set $v_1, v_2, v_3 \dots$ etc. against the corresponding column and set $u_1, u_2, u_3 \dots$ etc. against the corresponding row. Then determine a set of u_i and v_j such that for each occupied cell, using $u_i + v_j = c_{ij}$.

Step-3: Assign 0 to one of the u_i or v_j for which the corresponding row or column have the maximum number of individual allocations.

Step-4: Then find the cell evaluations (Subtract the above matrix cells from the corresponding cells of original matrix) $u_i + v_j$ for each unoccupied cell and enter at the middle of the corresponding unoccupied cell and encircle.

Step-5: Then we calculate d_{ij} (difference of each occupied cell) by the $d_{ij} = C_{ij} - (u_i + v_j)$ for each unoccupied cell and enter at the middle of the corresponding unoccupied cell and encircle.

Step-6: If all the d_{ij} is non-negative, then the basic feasible solution is optimal. On the other hand, if anyone of d_{ij} is $-ve$, then the basic feasible solution is not optimal.

Step-7: Select the largest negative value of d_{ij} . If there have more than one equal cell, then any one can be chosen. Then draw a closed loop for the unoccupied cell. Starting the closed loop with the largest negative value of d_{ij} and draw a closed loop with the occupied cells only.

Step-8: Mark the identified cell as $+ve$ and each occupied cell at the corners of the path alternatively $-ve, +ve, -ve, \dots$ and so on.

Step-9: check all the negative position and consider the smallest transportation cost that has been assigned a $-ve$ sign. Now, $+$ and $-$ demand and supply values of all the positions of $+$ and $-$.

Step-10: Repeat the whole procedure until the optimum solution is obtained.

The optimal solution of the problem using MODI method is, $X_{12} = 50, X_{14} = 25, X_{22} = 15, X_{23} = 45, X_{31} = 30, X_{33} = 10$.

Hence, the minimum transportation cost of the problem is, $50*22 + 25*5 + 15*31 + 45*14 + 30*5 + 10*4 = \text{Rs. } 2510$.

We solved one more TP along with the original one and kept the results in the below table to see the effectiveness of the EQM.

TP	NWCM (In Rs.)	EQM (In Rs.)	MODI In Rs.)
3X4 matrix	2785	2510	2510
4X3 matrix	640	595	595

VI. CONCLUSION

In this study, we proposed EQM for finding the IBFS to the transportation problem. It refers to choose the best distribution of cost and time from the all combinations. The EQM obtained the optimal solution or the closest to optimal solution with a minimum computation time. As well as, use of EQM reduces the complexity of the problems which is shown in the tables.

VII. REFERENCES

- Hitchcock F.L. (1941). The distribution of a product from several sources to numerous localities, *Journal of Mathematical Physics*, vol.20, (pp. 224-230).
- Koopmans T.C. (1949). Optimum Utilization of Transportation System, *Econometrica*, Supplement vol.17.
- Dantzig G.B. (1951). *Linear Programming and Extensions*, Princeton University Press, Princeton.
- Elaine L.Y. Man, and Adam B. (2011). A Qualitative Approach of Identifying Major Cost Influencing Factors in Palm Oil Mills and the Relations towards Production Cost of Crude Palm Oil, *American Journal of Applied Sciences*, 8, (pp. 441-446).
- Sudha S.V., and Thanushkodi k. (2012). A genetic based neuro fuzzy technique for process grain sized scheduling of parallel jobs, *Journal of Computer Science*, 8, (pp. 48-54)

- Amir S., Aashtiani H. Z., and Mohammadian K. A. (2009). A Short-term Management strategy for Improving transit network efficiency, *American Journal of Applied Sciences*, 6, (pp. 241-246).
- Norozi A., Ariffin M. K. A., and Ismail N. (2010). Application of Intelligence Based Genetic Algorithm for Job Sequencing Problem on Parallel Mixed-Model Assembly Line, *American Journal of Engineering and Applied Sciences*, 3, (pp. 15-24).
- Suganthi P., and Tamilarasi A. (2012). Impact of malicious nodes under different route refresh intervals in adhoc network, *American Journal of Applied Sciences*, 9, (pp. 18-23).
- Hakim M. A. (2012). An Alternative Method to Find Initial Basic Feasible Solution of a Transportation Problem, *Annals of Pure and Applied Mathematics*, Vol. 1, 2, (pp. 203-209).
- Goyal S. K. (1984). Improving VAM for unbalanced transportation problems, *Journal of Operational Research Society*, 35(12), (pp. 1113-1114).
- Gupta P. K., and Man Mohan. (1993). *Linear Programming and Theory of Games*, 7th edition, Sultan Chand & Sons, New Delhi, (pp. 285-318).
- Patel S. M. (1992). Resolution of Closed Loop in Transportation Problem, *International Journal of Management and Systems*, Vol. 81, 1, (pp. 35-46).
- Rao S. S. (1987). *Optimization Theory and Applications*, Wiley Eastern Limited.
- Sen N. et al. (2008a, 2008b, 2008c). Mathematical Modeling of Transportation Related Problem of Southern Assam and Its Optimal Solution, *AUJ Science*, Vol. 31, 1, (pp. 22-27).